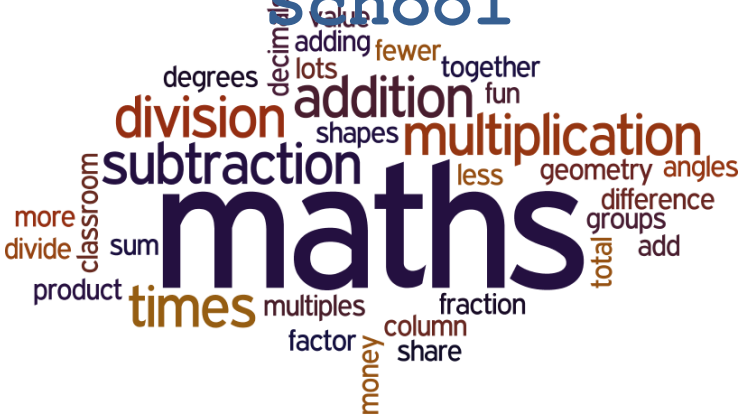
Calculation Policy of:

Wrockwardine Wood Infant School & Nursery

&

Wrockwardine Wood C.E. Junior School



Children learn by:

- doing it (concrete)
- remembering it (pictorial)
- seeing it (abstract)
- recording it (communication)

Date of policy: October 2023

Review date: April 2026

Contents

Videos to support learning:

The following link offers further materials and support in embedding the National Curriculum:

https://www.ncetm.org.uk/in-the-classroom/national-curriculum-resource-tool/

Key representations to support conceptual understanding of addition and subtraction.

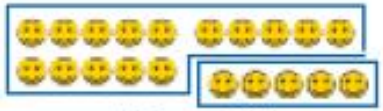
6 + 10 = 16
16 + 10 = 26
26 + 10 = 36
36 + 10 = 46

36 + 20 = 56

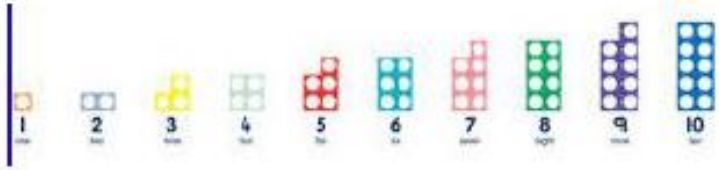
96 – 10 = 86
86 – 10 = 76
76 – 10 = 66
etc.

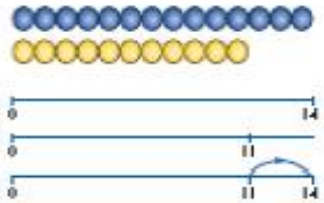
76 – 30 = 46

8 + ? = 10



15 + 5 = 20



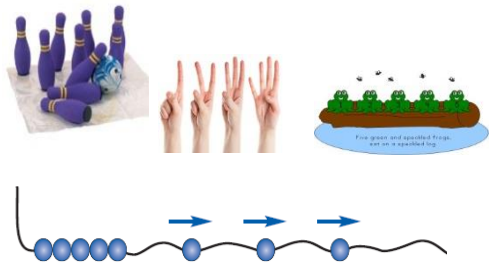
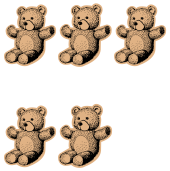


10 = 7 + 3

The difference between 11 and 14 is 3.
14 – 11 = 3
11 + □ = 14

Addition & Subtraction EYFS

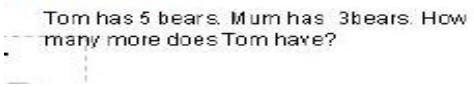
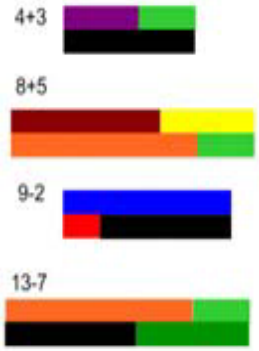
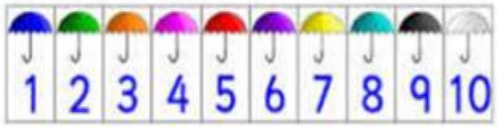
Statutory requirements	**Early Learning Goal - Number** **Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.**
Guidance	**Early practical experiences to include number rhymes, songs, stories and daily counting opportunities.** In practical activities and discussion, begin to use the vocabulary involved in addition and subtraction. Add, more, and, make, sum, total, altogether, score, double, one more, two more, ten more, how many more to make? How many more is…than…? Take away, leave, how many are left/left over? How many have gone? One less, two less, ten less, how many fewer is … than…?, difference between, is the same as • **Understand that the total gets bigger when something is added.** • **Add two single-digit numbers.** • **Understand that addition is commutative.**
Progression	During 30 -50 months the children have begun to graphically represent using fingers, marks on paper or pictures. They can compare two groups of objects, saying when they have the same number and have shown an interest in solving number problems. They can compare two groups of objects, saying when they have the same number. Within 40 -60 months the children relate addition to combining two groups and subtraction to *taking away,* finding the total number of items in two groups by counting all of them. In practical activities and discussion, they begin to use the vocabulary involved in adding and subtracting and record, using marks that they can interpret and explain. Working within the Early learning goal the children count reliably with numbers from one to 20, place them in order and say which

	number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.
	Addition and Subtraction
Representations to support mental and written calculations	How many would there be if 1 more duck swam over?   and  Count 5 objects into a bag. How many objects in the bag? Count 2 more objects into the bag. How many objects are in the bag now?   Jane had 3 bears. She was given 2 more. How many does she have now? 

	Addition	Subtraction
Mental Calculations	• Find the total number of items in two groups by counting all of them. • Say the number that is one more than a given number. • Partition a number in different ways and recombine to understand the total stays the same. e.g. 5 • Say the number which is one less than a given number. • Counting on, on fingers, orally, and number lines. • Make decisions about how to solve a problem	• Find one less from a group of up to five objects, then ten objects. • Remove objects from a small group and count how many are left. • Know that the answer gets smaller when objects are taken away. • Say the number which is one less than a given number. • Counting back on fingers, orally, and number lines. • Make decisions about how to solve a problem
Written Calculations	Writing numerals. Record using marks and pictures they can interpret and explain.	

Addition & Subtraction Year 1	
Statutory requirements	**Pupils should be taught to:** **□□read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs** **□□represent and use number bonds and related subtraction facts within 20** **□□add and subtract one-digit and two-digit numbers to 20, including zero** **□□solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = □ – 9.**
Guidance	Pupils memorise and reason with number bonds to 10 and 20 in several forms (for example, 9 + 7 = 16; 16 – 7 = 9; 7 = 16 – 9). They should realise the effect of adding or subtracting zero. This establishes addition and subtraction as related operations. Pupils combine and increase numbers, counting forwards and backwards. They discuss and solve problems in familiar practical contexts, including using quantities. Problems should include the terms: put together, add,

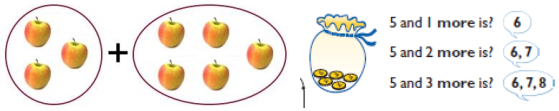
	altogether, total, take away, distance between, difference between, more than and less than, so that pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.
Progression	During the Foundation Stage, children related addition to combining two groups and subtraction to *taking away*, engaging *in* practical activities. In Year 1, children use mathematical statements to record addition and subtraction. They read, interpret and write the symbols +, – and =. Through practice of addition and subtraction, children learn the number trios for numbers to 20 (8 + 5 = 13, 13 – 8 = 5, 13 – 5 = 8). They use different strategies to help them derive number facts, such as adding numbers in any order, or finding a difference by counting up.

	Addition	**Subtraction**
Representations to support mental and written calculations	      	 

Addition and Subtraction

Counting and Combining sets of Objects

Combining two sets of objects (aggregation) which will progress onto adding on to a set (augmentation)



Understanding of counting on with a number track.

Understanding of counting on with a numberline (supported by models and images).

Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10.

Recognise place value of numbers to and beyond 20

Use bundles of straws, numicon and Dienes to model partitioning teen numbers into tens and ones and develop an understanding of place value.

Understand subtraction as take-away.

Taking away objects from a set and counting how many are left using real objects.

Understanding of counting back with a number track and a number line.

Regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10.

Progress from using number lines with every number shown to number lines with significant numbers shown.

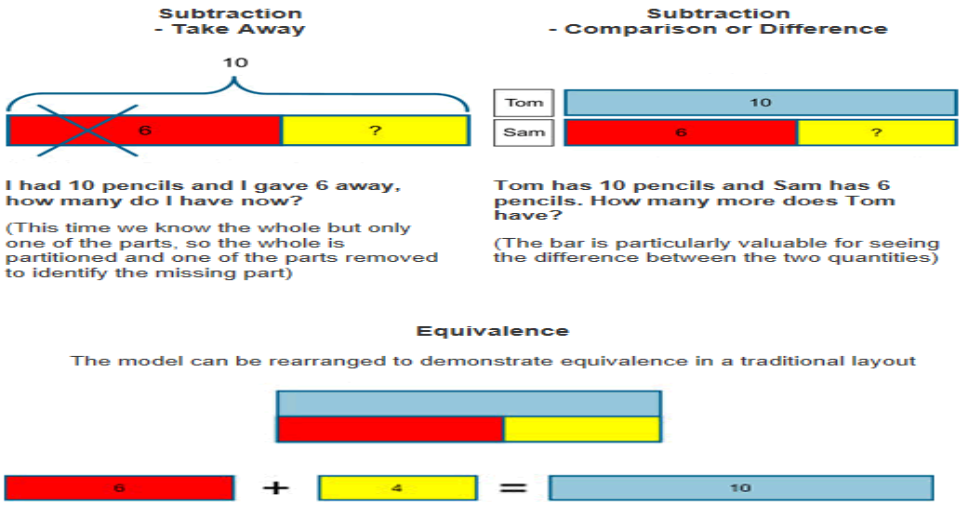
Understand subtraction as finding the difference.

To be introduced with concrete objects which children can move (including cards with pictures) before progressing to pictorial representation.

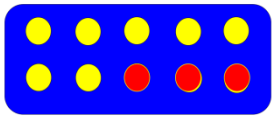
Find small differences by counting on.

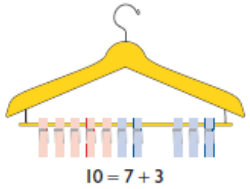
Understand the relationship and links between addition and subtraction

Children to see the equals sign as signifying equality. They should become used to seeing it in different positions



Compare (what's the same/different?)for commutative sums e.g. 3+7=7+3
Memorise and reason with number bonds for numbers to 20, experiencing the = sign in different positions.
See addition and subtraction as related operations. E.g. 7 + 3 = 10 is related to 10 – 3 = 7, understanding of which is supported by images:





e.g. 7 = 6 + 1, 7 = 5 + 2, 7 = 4 + 3

Work with all numbers up to 20

If we know 4 + 5 = 9 We also know: ,
5 + 4 = 9 9 - 5 = 4 9 - 4 = 5 14 + 5 = 19 19 - 14 = 5, etc

<table>
<tr><td></td><td colspan="2"><u>Solve one-step problems that involve addition and subtraction,</u></td></tr>
<tr><td>Mental Calculations</td><td><u>Vocabulary</u>
Addition, add, forwards, put together, more than, total, altogether, distance between, difference between, equals = same as, most, pattern, odd, even, digit, counting on.
<u>Generalisations</u>
• True or false? Addition makes numbers bigger.
• True or false? You can add numbers in any order and still get the same answer.
<u>Key Questions</u>
How many altogether? How many more to make…? I add …more. What is the total? How many more is… than…? How much more is…? One more, two more, ten more…
What can you see here? Is this true or false? What is the same? What</td><td><u>Vocabulary</u>
Subtraction, subtract, take away, distance between, difference between, more than, minus, less than, equals = same as, most, least, pattern, odd, even, digit,

<u>Generalisations</u>
True or false? Subtraction makes numbers smaller

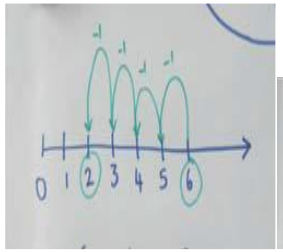
<u>Some Key Questions</u>
How many more to make…? How many more is… than…? How much more is…? How many are left/left over? How many have gone? One less, two less, ten less… How many fewer is… than…? How much less is…?
What can you see here?
Is this true or false?</td></tr>
<tr><td>Written Calculations</td><td colspan="2"><u>Graphic Representation</u> <u>+ = signs and missing numbers</u> <u>Solve one-step problems that involve addition and subtraction,</u>
Children to understand the concept of equality before using the ‘=’ sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as ‘the answer’. They should become used to seeing the = sign in in different positions.
e.g. 7 = □ - 9; 20 - □ = 9; 15 – 9 = □; □ - □ = 11; 16 – 0 = □
2 = 1+ 1</td></tr>
</table>

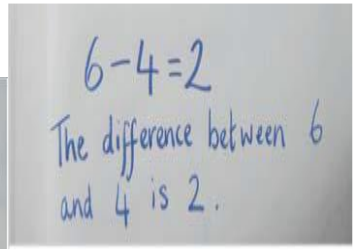
2 + 3 = 4 + 1

Missing numbers to be placed in all possible places.

3+ 4 = □	□ = 3 + 4	7 - 3 = □	□ - 3 =
3+ □ = 7	7 = □ + 4	7 - □ = 4	17 – 13 □

Subtract one digit and two digit numbers to 20, including zero.



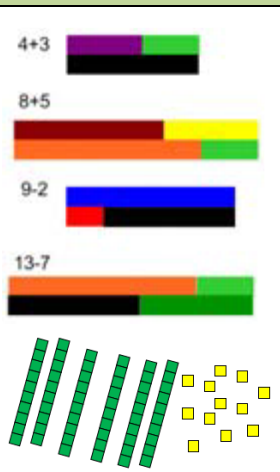
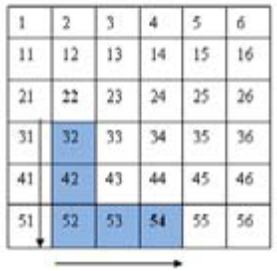


Addition & Subtraction Year 2	

Statutory requirements	**Pupils should be taught to:**
	• **solve problems with addition and subtraction:**
	• **use concrete objects and pictorial representations, including those involving numbers, quantities and measures**
	• **apply their increasing knowledge of mental and written methods**
	• **recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100**
	• **add and subtract numbers using concrete objects, pictorial representations, and mentally, including:**
	  ○ **a two-digit number and ones**
	  ○ **a two-digit number and tens**
	  ○ **two two-digit numbers**
	  ○ **adding three one-digit numbers**
	• **show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot**
	• **recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.**

Guidance	Pupils extend their understanding of the language of addition and subtraction to include sum and difference. Pupils practise addition and subtraction to 20 to become increasingly fluent in deriving facts such as using 3 + 7 = 10; 10 – 7 = 3 and 7 = 10 – 3 to calculate 30 + 70 = 100; 100 – 70 = 30 and 70 = 100 – 30. They check their calculations, including by adding to check subtraction and adding numbers in a different order to check addition (for example, 5 + 2 + 1 = 1 + 5 + 2 = 1 + 2 + 5). This establishes commutativity and associativity of addition.

	Recording addition and subtraction in columns supports place value and prepares for formal written methods with larger numbers.
Progression	Children use mental methods to solve problems using addition and subtraction, as well as using objects and pictorial representations. They begin to record addition and subtraction in columns, reinforcing their knowledge of place value. They independently use addition and subtraction facts to 20, and this helps them derive number facts up to 100, such as seeing the parallels between 2 + 6 = 8 and 20 + 60 = 80. They add and subtract different combinations of numbers, including two two-digit numbers. They understand the inverse relationship between addition and subtraction (that one operation undoes the other), and use this to check their calculations.

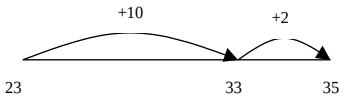
	Addition	**Subtraction**
Representations to support mental and written calculations	  	 

	Addition	Subtraction
Mental Calculations And Jottings	**Count on regularly in steps of 2, 3, 5 and 10.** **Counting on in tens from any number, leading to adding of multiples of 10.** Practise addition to 20 to become increasingly fluent. They should use concrete objects such as bead strings and number lines to explore missing numbers 45 + ___ = 50. **Using known facts to derive others** If I know: 2+3 = 5 I also know: 3+2 = 5 20 + 30 = 50 30 + 20 = 50 50—30 = 20 50—20 = 30 **Using the bar model** Continue to use the bar model, as well as images in the context of **measures.** **Missing number problems** e.g 14 + 5 = 10 + □ 32 + □ + □ = 100 35 = 1 + □ + 5 e.g. 52 − 8 = □; □ − 20 = 25; 22 = □ − 21; 6 + □ + 3 = 11	**Count back regularly, in steps of 2, 3, 5 and 10.** **Count back in tens from any number, leading to subtracting multiples of 10.** Practise subtraction to 20 to become increasingly fluent. **Using known facts to derive others** If I know: 2+3 = 5 I also know: 3+2 = 5 20 + 30 = 50 30 + 20 = 50 50—30 = 20 50—20 = 30 **Using the bar model** Continue to use the bar model, as well as images in the context of **measures.** **Missing number problems** e.g 14 + 5 = 10 + □ 32 + □ + □ = 100 35 = 1 + □ + 5 e.g. 52 − 8 = □; □ − 20 = 25; 22 = □ − 21; 6 + □ + 3 = 11 Use a range of representations (also see Y1).

Use a range of representations (also see Y1).

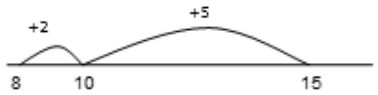
Continue to use number lines to develop understanding of:
Counting on in tens and ones

23 + 12 = 23 + 10 + 2
= 33 + 2
= 35



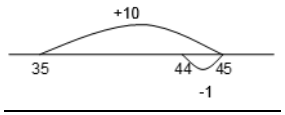
Partition and bridge through 10.

The steps in addition often bridge through a multiple of 10
e.g. Children should be able to partition the 7 to relate adding the 2 and then the 5.
8 + 7 = 15



Add 9 or 11 by adding 10 and adjusting by 1

Add 9 by adding 10 and adjusting by 1
35 + 9 = 44

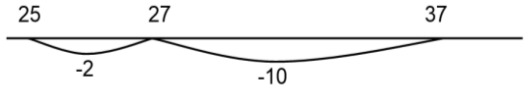


Practical partioning of 2 digit numbers

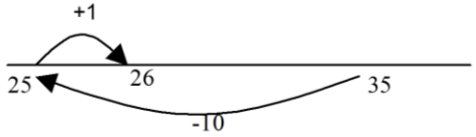
Bundles of straws or dienes to represent and partition 2 digit numbers.

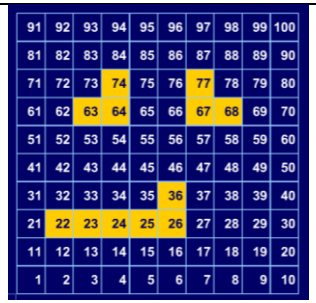
Continue to use number lines to develop understanding of:
Counting back in tens and ones , take-away and difference.

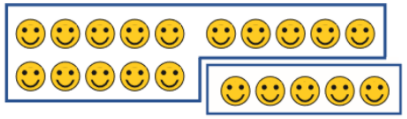
37 – 12 = 37 – 10 – 2
= 27 -2
=25

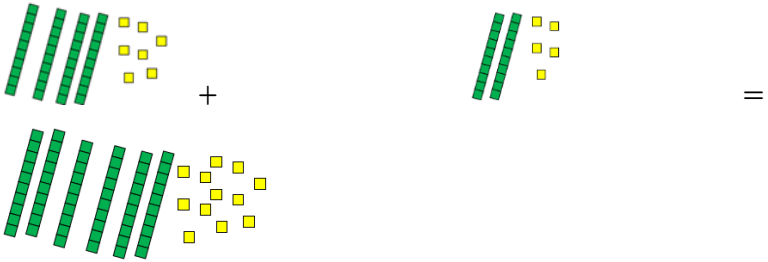
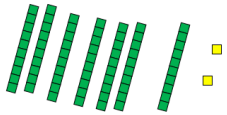


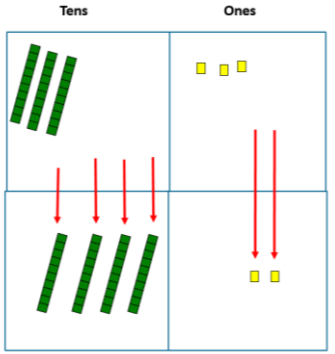
Subtract 9 or 11 by adjusting



	Use a range of number squares to explore patterns in calculations74 +11, 77 + 9 encouraging children to think about 'What do you notice?' where partitioning or adjusting is used. Learn to check their calculations, by using the inverse. Continue to see addition as both combining groups and counting on. Use structured apparatus to model partitioning into tens and ones and learn to partition numbers in different ways e.g. 23 = 20 + 3 = 10 + 13. **Vocabulary** +, add, addition, more, plus, make, sum, total, altogether, how many more to make…? how many more is… than…? how much more is…? =, equals, sign, is the same as, Tens, ones, partition Near multiple of 10, tens boundary, More than, one more, two more… ten more… one hundred more **Generalisation**	 **Use a range of number squares to** model calculations such as 74 – 11, 77 – 9 or 36 – 14, where partitioning or adjusting are used. Learn to check their calculations, by using the inverse. Continue to see subtraction as both take away and finding the difference, and should find a small difference by counting up. Use structured apparatus to model partitioning into tens and ones and learn to partition numbers in different ways. **Vocabulary** Subtraction, subtract, take away, difference, difference between, minus Tens, ones, partition Near multiple of 10, tens boundary

	• Noticing what happens when you count in tens (the digits in the ones column stay the same) • Odd + odd = even; odd + even = odd; etc • show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot • Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems. **Some Key Questions** How many altogether? How many more to make…? How many more is… than…? How much more is…? Is this true or false? If I know that 17 + 2 = 19, what else do I know? (e.g. 2 + 17 = 19; 19 – 17 = 2; 19 – 2 = 17; 190 – 20 = 170 etc). What do you notice? What patterns can you see?	Less than, one less, two less… ten less… one hundred less More, one more, two more... ten more... one hundred more **Generalisation** • Noticing what happens when you count in tens (the digits in the ones column stay the same) • Odd – odd = even; odd – even = odd; etc • show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot • Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems. This understanding could be supported by images such as this.  15 + 5 = 20 **Some Key Questions** How many more to make…? How many more is… than…? How

		much more is…? How many are left/left over? How many fewer is… than…? How much less is…? Is this true or false? If I know that 7 + 2 = 9, what else do I know? (e.g. 2 + 7 = 9; 9 – 7 = 2; 9 – 2 = 7; 90 – 20 = 70 etc). What do you notice? What patterns can you see?
	Towards a Written Method Partitioning in different ways and recombining 47+25 **47** **25** **60+12**  Leading to exchanging: 	**Towards a Written Method** Partioining to subtract.using structured apparatus. 75 – 42 Recording addition and subtraction in expanded columns can support understanding of the quantity aspect of place value and prepare for efficient written methods with larger numbers. The numbers may be represented with Dienes apparatus. E.g. 75 – 42

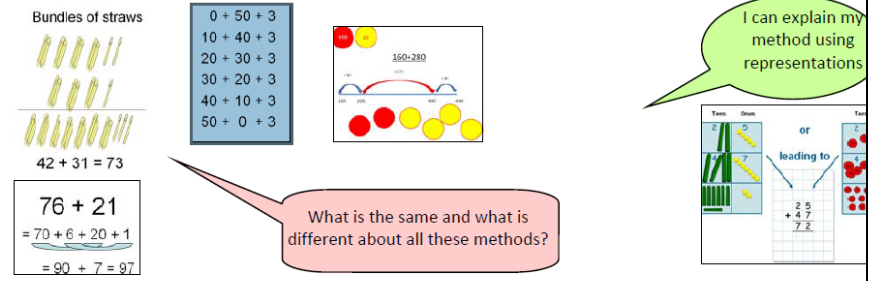
		 70 5 -40 2 30 3 Use suitable resources as required (See models and images page). Children that have not achieved the age related expectations for Year 2 should not move onto formal written methods until they are secure.
Written Calculations	**Expanded written method** 40 + 7 + 20 + 5 = 40+20 + 7 + 5 = 60 + 12 = 72 40 + 7 + 20 + 5 60 + 12 = 72	Informal methods to support written subtraction calculations Subtract(without decomposition) using partitioning and equipment, e.g. 37−12 = 37−10−2 =27−2 =25

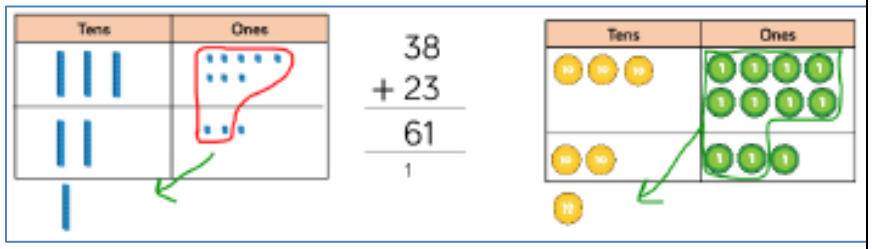
Addition & Subtraction Year 3	
Statutory requirements	Pupils should be taught to: ▪ add and subtract numbers mentally, including: ▪ a three-digit number and ones ▪ a three-digit number and tens ▪ a three-digit number and hundreds ▪ add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction ▪ estimate the answer to a calculation and use inverse operations to check answers ▪ solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.
Guidance	Pupils practise solving varied addition and subtraction questions. For mental calculations with two-digit numbers, the answers could exceed 100. Pupils use their understanding of place value and partitioning, and practise using columnar addition and subtraction with increasingly large numbers up to three digits to become fluent (see Mathematics Appendix 1).
Progression	In Year 3, children practise mentally adding and subtracting combinations of numbers, including three-digit numbers. When using written methods for addition and subtraction, children learn to write the digits in columns, using their knowledge of place value to align the digits correctly. Children begin to use estimation to work out the rough answer to calculations in advance, and use inverse operations to check their final answers – for example, checking 312 + 43 = 355 by working out 355 – 43 = 312.

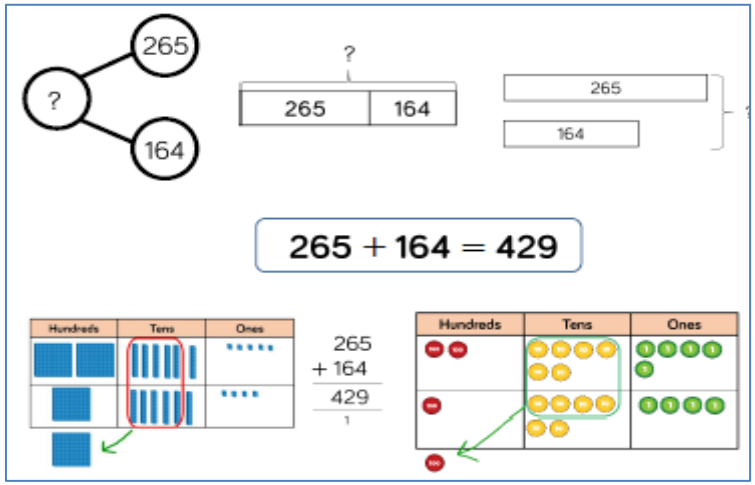
	● Children should practise solving varied addition and subtraction questions. For mental calculations with two-digit numbers, the answers could exceed 100. ● Children should use their understanding of place value and partitioning, and practise using columnar addition and subtraction with increasingly large numbers up to three digits to become fluent (see National Curriculum Appendix 1).	
	Addition	**Subtraction**

Representations to support mental and written calculations

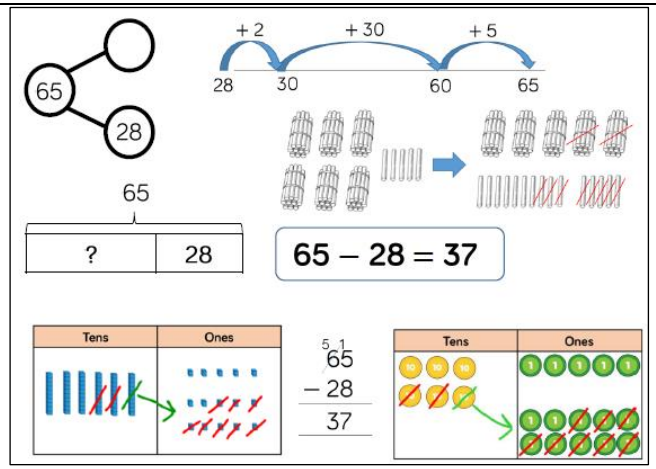
Use a range of concrete, pictorial and abstract representations, including those below

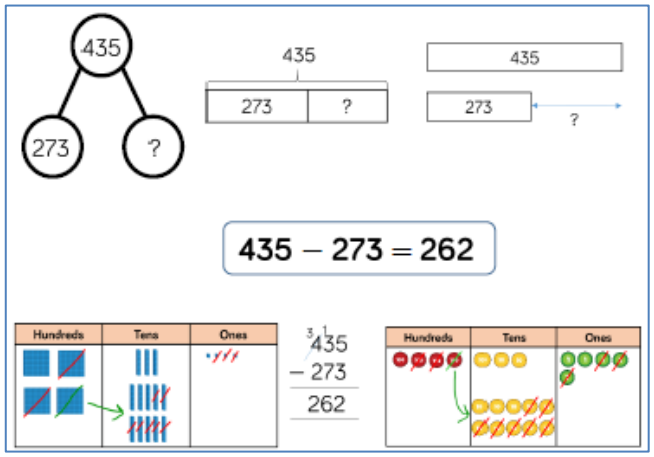


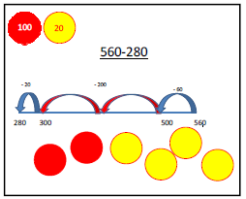




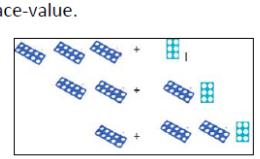
- Base 10 and place value counters are the most effective manipulatives when







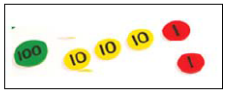
Partitioning and re- partitioning support the understanding of place-value.



All of these representations still comprise the amount of 36.

Introduce transition from concrete place value representations, (e.g. dienes or straws), to pictorial – such as place value counters or money.

132 in dienes

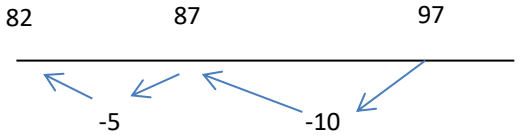
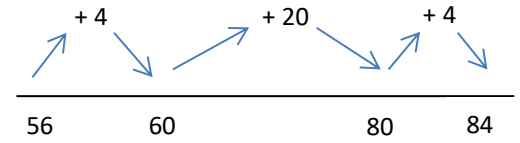


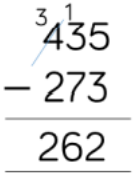
132 in place value counters.

Revert to concrete manipulatives and expanded methods whenever difficulties aris

	adding numbers up to 3 digits. • Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.	

	Addition		**Subtraction**
Mental Calculations	Add and subtract mentally, including:	Common mental calculation strategies:	Use known number facts and place value to subtract

	▪ a three-digit number and ones ▪ a three-digit number and tens ▪ a three-digit number and hundreds	▪ partitioning and recombining ▪ doubles and near-doubles ▪ use number pairs to 10 and 100 ▪ adding near multiples of ten and adjusting ▪ using patterns of similar calculations ▪ using known number facts ▪ bridging through ten, hundred ▪ complementary addition	Continue as in Year but with appropriate numbers, e.g. 97 – 15 = 72  With practise, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up and back is the more efficient method for calculations, such as 57 – 12, 86 – 77 or 43 – 28 **Complementary addition** 84 – 56 = 28 
Written Calculations	Add numbers with up to three-digits, using formal written (columnar) methods.	Subtract numbers with up to three digits, using formal written methods of columnar subtraction.	

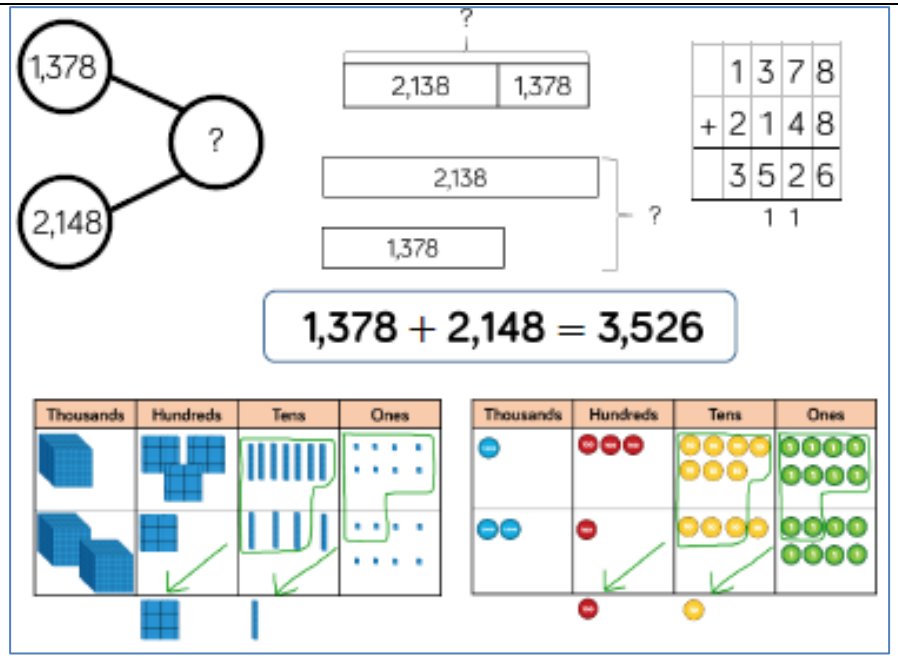
	Partition all numbers and recombine, starting with TU + TU, then HTU + TU, e.g 247 + 125 = 247 + 100 + 20+ 5 = 347 + 20 + 5 = 367 + 5 = 372 265 + 164 429 1 • Add to three digits, using physical and abstract representations (e.g. straws, dienes, place value counters, empty number lines) • Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.	
Vocabulary	Addend, aggregation, commutative, complement, sum Hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, exchange, number to one thousand. **See also Y1 and Y2**	Minuend, subtrahend, Hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange **See also Y1 and Y2**

Generalisations	Noticing what happens to the digits when you count in tens and hundreds. Odd + odd = even etc (see Year 2) Inverses and related facts – develop fluency in finding related addition and subtraction facts. Develop the knowledge that the inverse relationship can be used as a checking method. **Key Questions** What do you notice? What patterns can you see? When comparing two methods alongside each other: What's the same? What's different?	Noticing what happens to the digits when you count in tens and hundreds. Odd – odd = even etc (see Year 2) Inverses and related facts – develop fluency in finding related addition and subtraction facts. Develop the knowledge that the inverse relationship can be used as a checking method. **Key Questions** What do you notice? What patterns can you see? When comparing two methods alongside each other: What's the same? What's different?

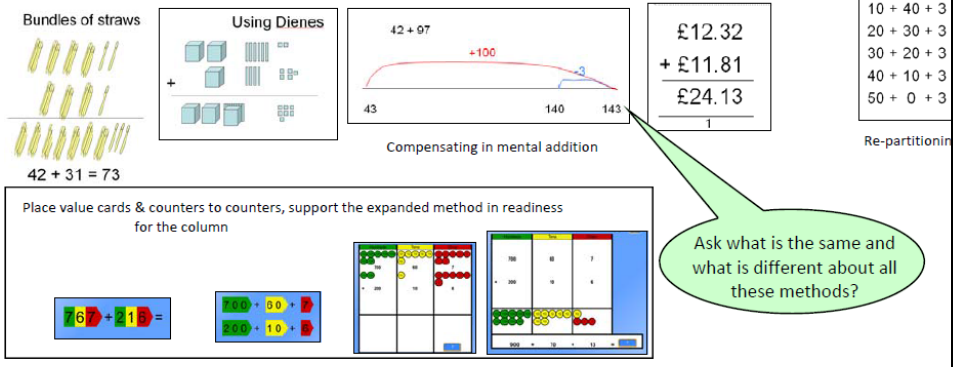
Addition & Subtraction Year 4	
Statutory requirements	Pupils should be taught to: ▪ add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate ▪ estimate and use inverse operations to check answers to a calculation

	▪ solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.	
Guidance	Pupils continue to practise both mental methods and columnar addition and subtraction with increasingly large numbers to aid fluency (see Mathematics Appendix 1).	
Progression	Children extend previous years' work by adding and subtracting numbers with up to four digits, using mental and written methods, including columnar addition and subtraction. They keep practising mental methods of addition and subtraction as well as written methods, performing calculations increasingly quickly and confidently. They continue using estimation as well as inverse operations to help check answers. Children should continue to practise both mental methods and columnar addition and subtraction with increasingly large numbers to aid fluency	
	Addition	**Subtraction**

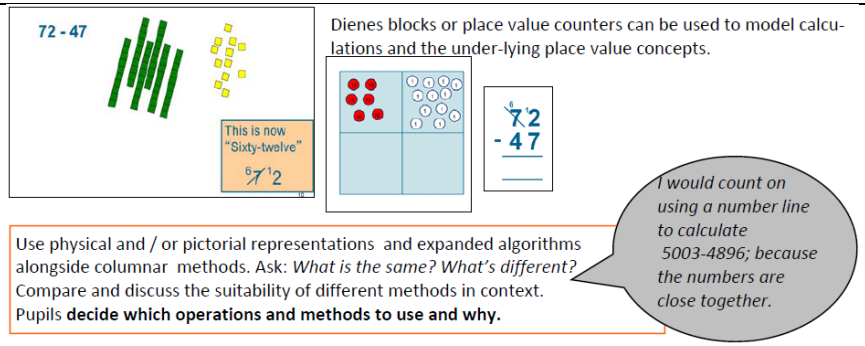
Representations to support mental and written calculations

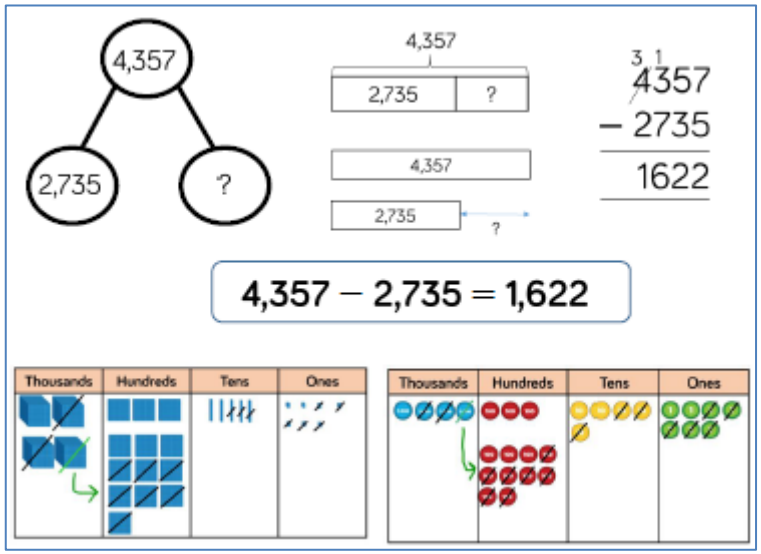


Use physical/pictorial representations alongside expanded and columnar methods.



- Base 10 and place value counters are the most effective manipulatives when adding numbers up to 4 digits.



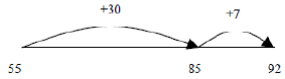


- Base 10 and place value counters are the most effective manipulatives when

	• Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.	subtracting numbers up to 4 digits. • Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method

	Addition	**Subtraction**

<table>
<tr>
<td>Mental Calculations</td>
<td>
Practise mental methods with increasingly large numbers

Consolidate partitioning and re-partitioning
Use compensation for adding too much/little and adjusting
Use straws, Dienes, place value counters,
empty number lines etc.


I know that 63 + 29 is the same as 63 + 30 –1

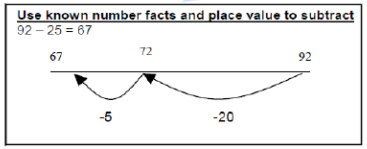
55 + 37 = 55 + 30 + 7
= 85 + 7
= 92

Common mental calculation strategies:
Partitioning and recombining
Doubles and near doubles
Use number pairs to 10 and 100
Adding near multiples of ten and adjusting
Using patterns of similar calculations
Using known number facts
Bridging though ten, hundred
Complementary addition

Continue to practise mental methods with increasingly large numbers to aid fluency. (From Non– Statutory Guidance).
Methods to support fluent calculation and encourage efficiency of method:
• Find a small difference by counting up.
E.g. 5003—4996
• Subtract nearest multiple of ten and adjust.
• Partition larger numbers

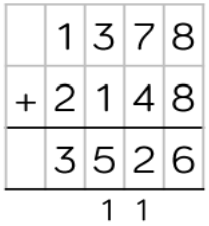
This could be done using an empty number line. Children should recall and use number facts to reduce the number of steps.

Whenever possible, children should be encouraged to visualise number lines and other basic, supporting representations to promote fluent work without jottings.

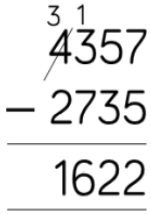


</td>
</tr>
<tr>
<td>Written Calculations</td>
<td>
Add numbers with up to four digits, using the formal written (columnar) method

Add three digit numbers using columnar method and then move onto 4 digits.
Include decimal addition for money



</td>
<td>
Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction.

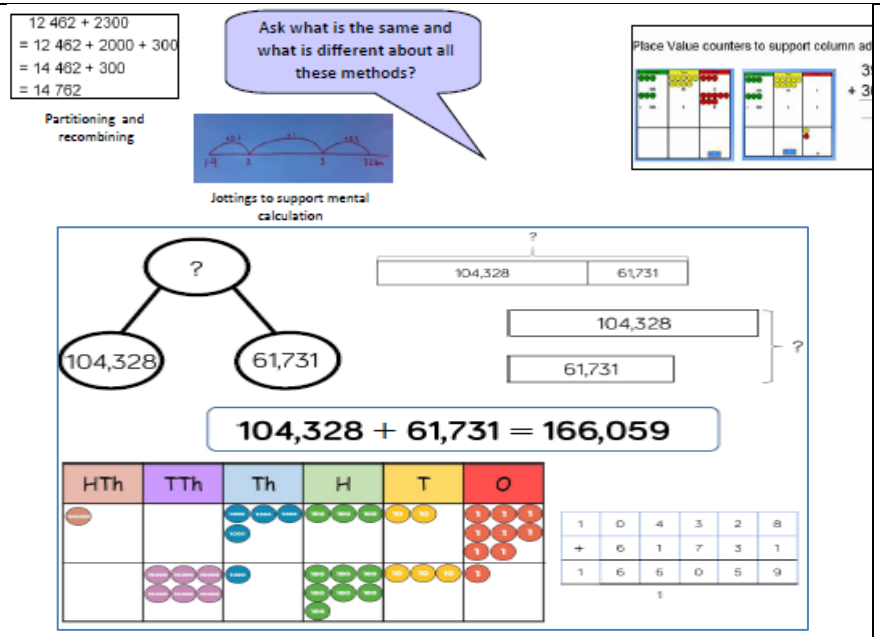
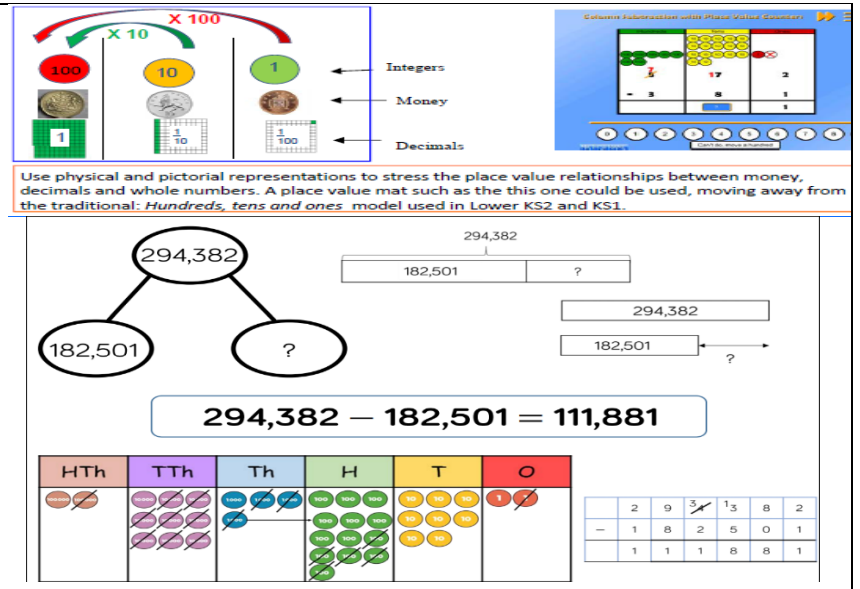


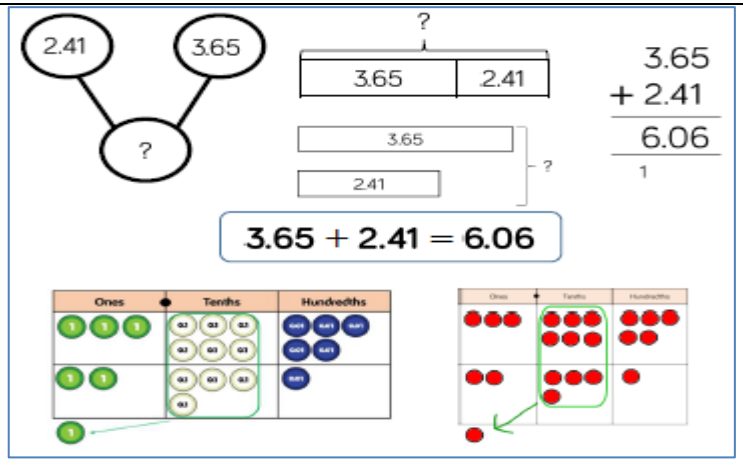
</td>
</tr>
<tr>
<td><u>Vocabulary</u></td>
<td colspan="2">add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make..? how much more? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? Equals sign, is the same as, negative integers.
Also see Yr. 1, 2 and 3 vocabulary</td>
</tr>
<tr>
<td><u>Generalisations</u></td>
<td colspan="2">Investigate when re-ordering works as a strategy for subtraction. Eg. 20 – 3 – 10 = 20 – 10 – 3, but 3 – 20 – 10 would give a different answer.

<u>Some Key Questions</u>
What do you notice?
What’s the same? What’s different?
Can you convince me?
How do you know?</td>
</tr>
</table>

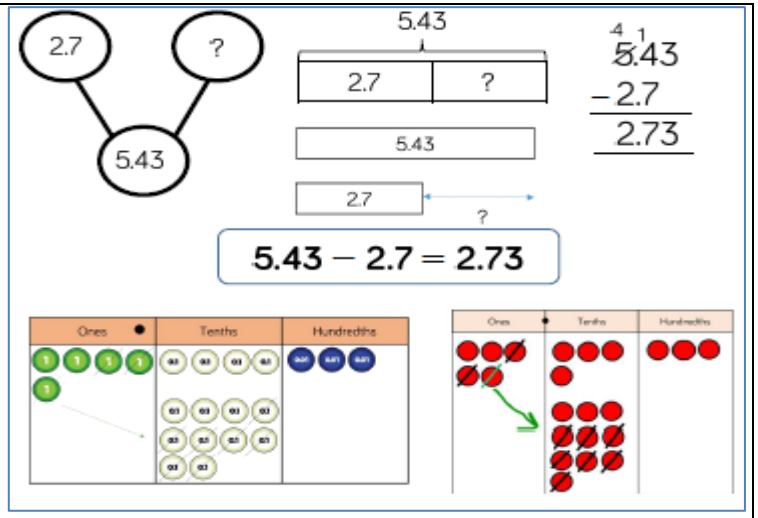
Addition & Subtraction Year 5

Statutory requirements	Pupils should be taught to: ▪ add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) ▪ add and subtract numbers mentally with increasingly large numbers ▪ use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy ▪ solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.	
Guidance	Pupils practise using the formal written methods of columnar addition and subtraction with increasingly large numbers to aid fluency (see Mathematics Appendix 1). They practise mental calculations with increasingly large numbers to aid fluency (for example, 12 462 – 2300 = 10 162).	
Progression	Children use columns in written addition and subtraction, accurately adding and subtracting numbers with more than four digits. They use mental methods to add and subtract increasingly large numbers, and use rounding to check their answers. With support they choose appropriate operations and methods, and work out the level of accuracy required to answer a particular problem. They will continue to develop this work in Year 6. Children should practise using the formal written methods of columnar addition and subtraction with increasingly large numbers to aid fluency. They should practise mental calculations with increasingly large numbers to aid fluency	
	Addition	**Subtraction**

Representations to support mental and written calculations	 • Place value counters or plain counters on a place value grid are most effective concrete resources when adding numbers with more than 4 digits. • At this stage, children should be encouraged to work in the abstract, using the column method to add large numbers efficiently.	 • Place value counters or plain counters on a place value grid are most effective concrete resources when subtracting numbers with more than 4 digits. • At this stage, children should be encouraged to work in the abstract, using the column method to subtract large numbers efficiently.

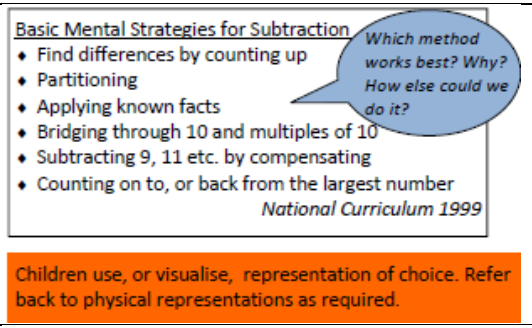
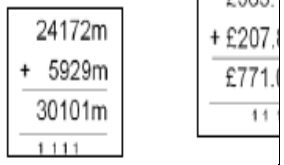
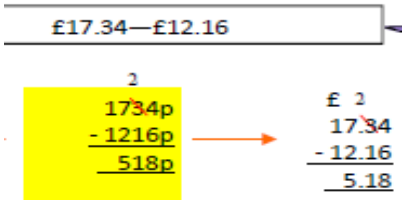


- Place value counters and plain counters on a place value grid are the most effective manipulatives when adding decimals with 1, 2 and then 3 decimal places.
- Ensure children have experience of adding decimals with a variety of decimal places. This includes putting this into context when adding money and other measures.

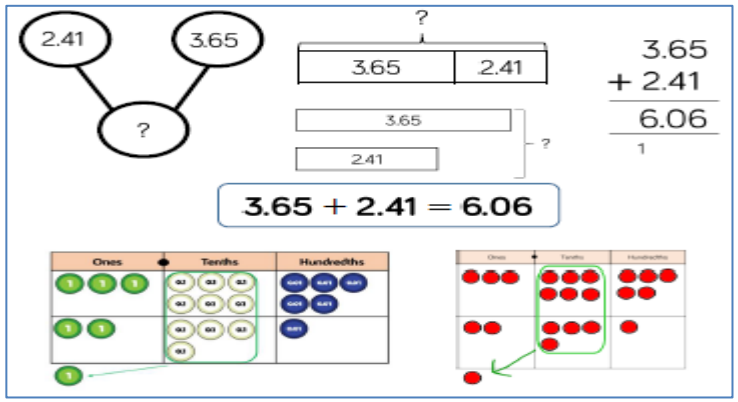
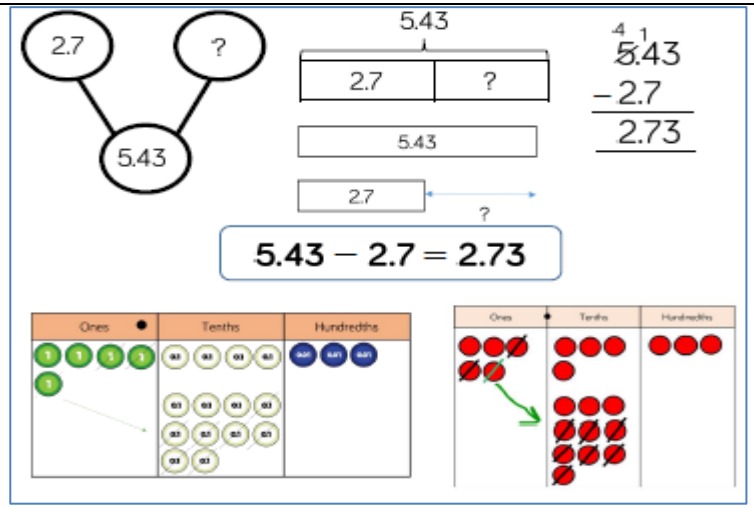


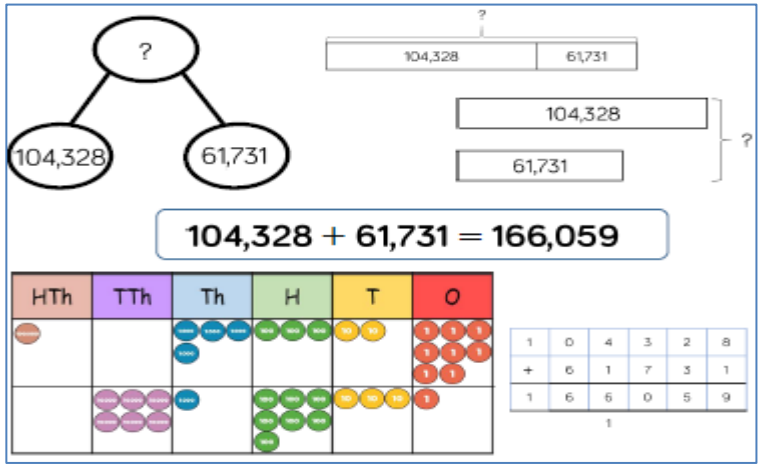
- Place value counters and plain counters on a place value grid are the most effective manipulatives when subtracting decimals with 1, 2 and then 3 decimal places.
- Ensure children have experience of subtracting decimals with a variety of decimal places. This includes putting this into context when adding money and other measures.

	Addition	Subtraction
Mental Calculations	• **Add numbers mentally with increasingly large numbers, e.g. 12 462 + 2300 = 14 762** • **Mentally add tenths, and one-digit numbers and tenths** • **Add decimals, including a mix of whole numbers and decimals, decimals with different nu of places, and complements of 1 (e.g. 0.83 + 0.17 = 1)** Children use representation of choice Refer back to pictorial and physical representations when needed. **Common mental calculation strategie** Partitioning and recombining Doubles and near doubles Use number pairs to 10 and 100 Adding near multiples of ten and adju Using patterns of similar calculations Using known number facts Bridging though ten, hundred, tenth Complementary addition	• Subtract numbers mentally with increasingly large numbers. E.g. 12 462 – 2300 = 10 162 • Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy . • Pupils practise adding and subtracting decimals, including a mix of whole numbers and decimals, decimals with different numbers of decimal places, and complements of 1 (for example, 1 - 0.17 = 0.83). • Pupils mentally add and subtract tenths, and one-digit whole numbers and tenths.

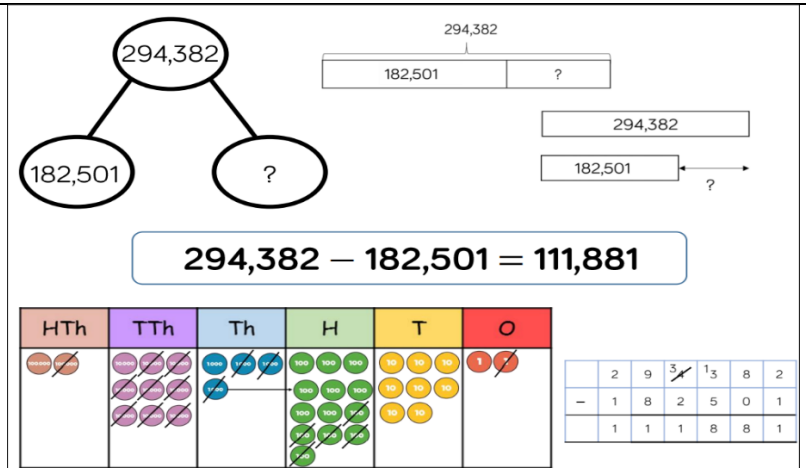
		
Written Calculations	Add whole numbers with more than four digits, using the formal written (columnar) method Add three digit numbers using columnar method and then move onto 4 digits. Include decimal addition for money 	**Add and subtract whole numbers with more than 4 digits, including using formal written me (columnar addition and subtraction).** ***(Pupils) practise adding and subtracting decimals.*** Begin with three-digit numbers using formal, columnar method; then move into four-digit nun 
Vocabulary	tens of thousands boundary, power of ten, efficient written method **Also see Yr. 1, 2, 3 and 4 vocabulary.**	
Generalisation	Sometimes, always or never true? The difference between a number and its reverse will be a multiple of 9. What do you notice about the differences between consecutive square numbers? Investigate a – b = (a-1) – (b-1) represented visually. **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?	

Addition & Subtraction Year 6		
Statutory requirements	solve problems involving addition, subtraction, multiplication and division use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.	
Guidance	Pupils practise addition, subtraction, multiplication and division for larger numbers, using the formal written methods of columnar addition and subtraction, short and long multiplication, and short and long division (see Mathematics Appendix 1). They undertake mental calculations with increasingly large numbers and more complex calculations. Pupils continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency. Pupils round answers to a specified degree of accuracy, for example, to the nearest 10, 20, 50 etc., but not to a specified number of significant figures. Pupils explore the order of operations using brackets; for example, 2 + 1 x 3 = 5 and (2 + 1) x 3 = 9. Common factors can be related to finding equivalent fractions.	
Progression	Children continue to practise using efficient written and mental methods for all four operations, working with larger numbers and increasingly complex calculations. Children should practise addition, subtraction, multiplication and division for larger numbers, using the efficient written methods of columnar addition and subtraction	
	Addition	**Subtraction**

Representations to support mental and written calculations	Use physical/pictorial representations alongside columnar methods where needed. Ask what is the same and what is different?  • Place value counters and plain counters on a place value grid are the most effective manipulatives when adding decimals with 1, 2 and then 3 decimal places. • Ensure children have experience of adding decimals with a variety of decimal places. This includes putting this into context when adding money and other measures.	 • Place value counters and plain counters on a place value grid are the most effective manipulatives when subtracting decimals with 1, 2 and then 3 decimal places. • Ensure children have experience of subtracting decimals with a variety of decimal places. This includes putting this into context when adding money and other measures.

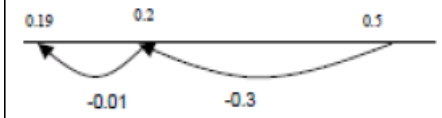
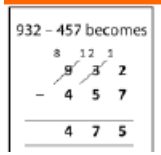
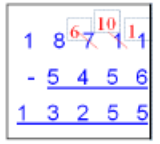
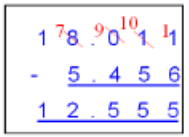


- Place value counters or plain counters on a place value grid are most effective concrete resources when adding numbers with more than 4 digits.
- At this stage, children should be encouraged to work in the abstract, using the column method to add large numbers efficiently.



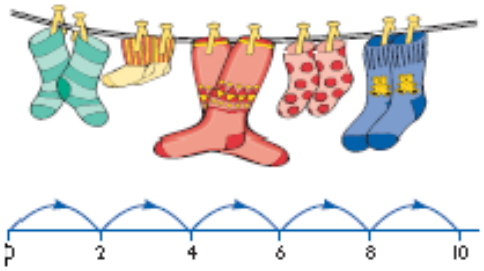
- Place value counters or plain counters on a place value grid are most effective concrete resources when subtracting numbers with more than 4 digits.
- At this stage, children should be encouraged to work in the abstract, using the column method to subtract large numbers efficiently.

	Addition	Subtraction
Mental Calculations	• Perform mental calculations, including with mixed operations and large numbers *(more complex calculations)* Children use representation of choice Consolidate partitioning and re-partitioning Use compensation for adding too much/little and adjusting Refer back to pictorial and physical representations when needed.	Children: • Perform mental calculations, including with mixed operations and large numbers. • Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy. • *They undertake mental calculations with increasingly large numbers and more complex calculations.*

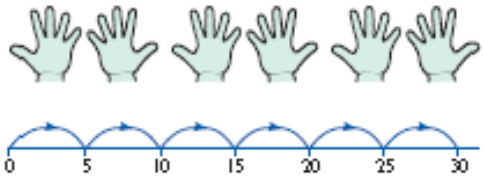
	Common mental calculation strategies: Partitioning and recombining Doubles and near doubles Use number pairs to 10 and 100 Adding near multiples of ten and adjusting Using patterns of similar calculations Using known number facts Bridging though ten, hundred, tenth Complementary addition	**Use known number facts and place value to subtract** 0.5 – 0.31 = 0.19 
Written Calculations	Add larger numbers using the formal written (columnar) method Add three digit numbers using columnar method and then move onto 4 digits. Include decimal addition for money. £563.14 + £207.88 £771.02 1 1 1 789 + 642 becomes 7 8 9 + 6 4 2 1 4 3 1 1 1 Answer: 1431	**Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction). Solve problems involving the calculation and conversions of units of measure, using decimal notation of up to three decimal places where appropriate. (MEASURES)** Move towards consolidation of formal, columnar method. For more complex calculations, with increasingly larger or smaller numbers, compare representations and expanded algorithms alongside columnar methods. Ask: What is the same? What's different? Compare and discuss the suitability of different methods, (mental or written), in context. Revert to expanded methods whenever difficulties arise  Consolidate columnar methods , paying particular attention to the occurrence of zeros as place holders.  
Vocabulary	Numbers to ten million, **See previous years**	

Generalisations	Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as BODMAS (Brackets, Order, Division, Multiplication, Addition, Subtraction), or could be encouraged to design their own ways of remembering. Sometimes, always or never true? Subtracting numbers makes them smaller. **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?

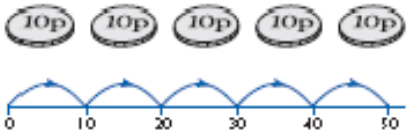
Key representations to support conceptual understanding of multiplication and division



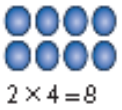
$2 + 2 + 2 + 2 + 2 = 10$
$2 \times 5 = 10$
2 multiplied by 5
5 pairs
5 hops of 2



$5 + 5 + 5 + 5 + 5 + 5 = 30$
$5 \times 6 = 30$
5 multiplied by 6
6 groups of 5
6 hops of 5



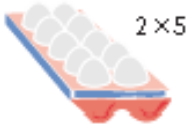
10 p + 10p + 10p + 10p + 10p = 50p
10 p × 5 = 50p
5 hops of 10



$4 \times 2 = 8$
$2 \times 4 = 8$

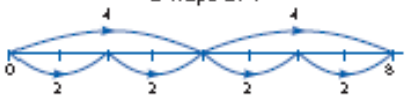
$5 \times 2 = 10$
$2 \times 5 = 10$

$2 \times 4 = 8$
$4 \times 2 = 8$



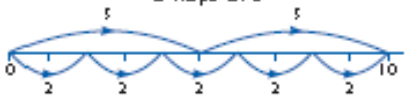
$2 \times 5 = 10$
$5 \times 2 = 10$

2 hops of 4

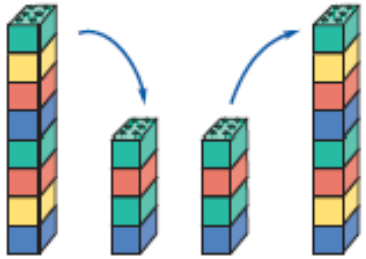


4 hops of 2

2 hops of 5

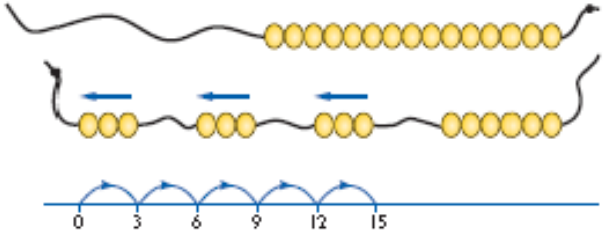


5 hops of 2



half of 8 is 4
$8 \div 2 = 4$

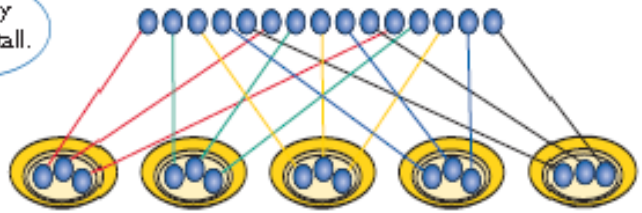
double 4 is 8
$4 \times 2 = 8$

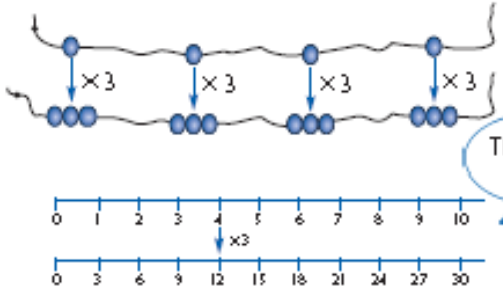


How many 3s in 15?

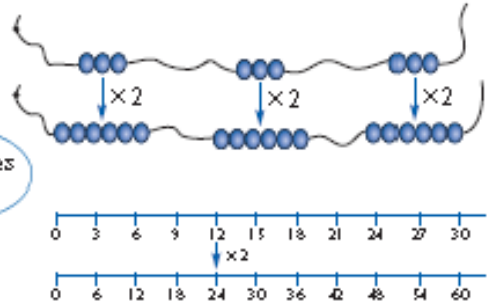
$15 \div 3 = 5$

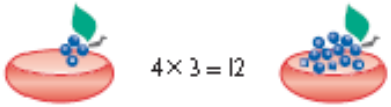
5 hops in 15. How big is each hop?
$15 \div 5 = 3$
15 shared between 5





Three times as many





$4 \times 3 = 12$

Twice as many

$12 \times 2 = 24$

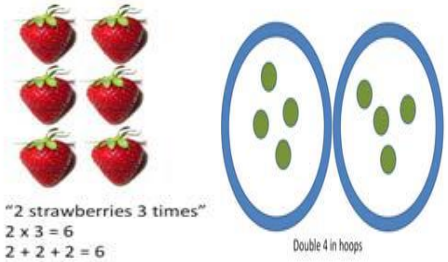
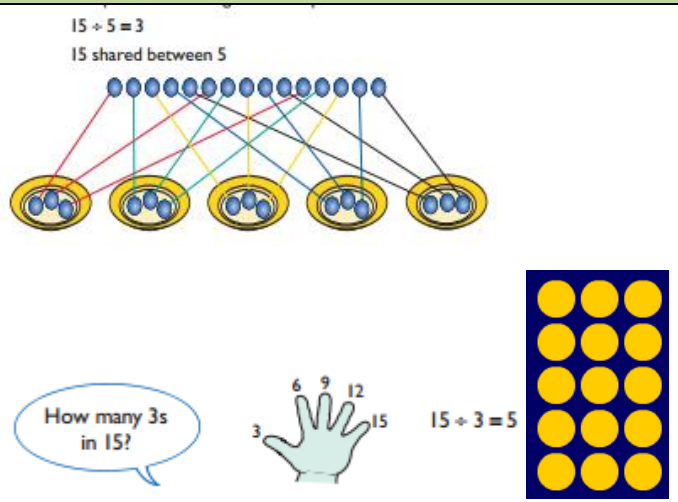
Multiplication & Division EYFS		
Statutory requirements	**Early Learning Goal - Number** **Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.**	
Guidance	**Early practical experiences to include number rhymes, songs, stories and daily counting/grouping opportunities.** In practical activities and discussion, use the vocabulary involved in multiplication: Sort, group, set, match, same, double, halve, groups of, sets of, lots of halve, share, share equally, one each, two each etc., group in pairs, left, left over.	
Progression	Within 30 -50 months the children have begun to graphically represent using fingers, marks on paper or pictures. They show an interest in solving number problems. They compare two groups of objects, saying when they have the same number and separate a group of three or four objects in different ways, beginning to recognise that the total is still the same. Within 40 -60 months they engage in practical activities and discussion, recording, using marks that they can interpret and explain. Working within the Early learning goal the children count reliably with numbers from one to 20, They solve problems, including doubling, halving and sharing.	
	Multiplication	**Division**
Representations to support mental and written calculations	Use a range of concrete and pictorial representations, including:	

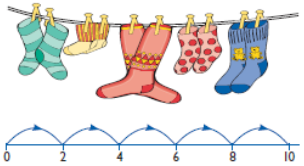
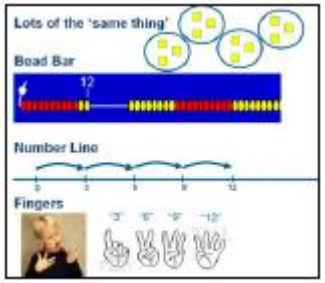
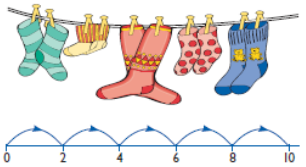
	    	

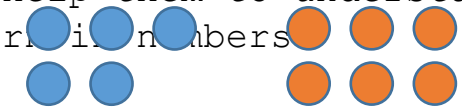
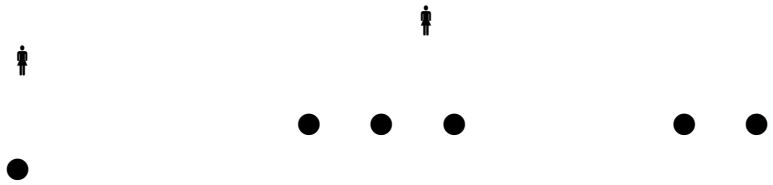
	Multiplication	Division

Mental Calculations	**Early practical experiences to include number rhymes, songs, stories and daily counting/grouping opportunities.** In practical activities and discussion, use the vocabulary involved in multiplication: Sort, group, set, match, same, double, halve, groups of, sets of, lots of. • **Sing rhymes using objects to model grouping in different ways.** • **Group objects in 2's.** • **Jump along number lines in jumps of 1 and 2.** *Start at 2 and jump 2 what happens?* • **Practical problems involving doubling**	**Early practical experiences to include number rhymes, songs, stories and daily counting opportunities.** In practical activities and discussion, use the vocabulary involved in division: Halve, share, share equally, one each, two each etc., group in pairs, left, left over. • **Make and compare sets/groups of objects saying when they have the same number.** • **Separate a group of up to 6 objects in different ways to recognise that the total is still the same** **Practical problems involving sharing and halving** • **Share in many practical contexts.** (Use cross curricular links) • **Understand the language of half.** *Many experiences of cutting and slicing objects, towers of cubes, pieces of paper into half to understand that the 2 halves have to be equal.* • **Solve practical problems involving halving.** *e.g. half of the 8 biscuits have gone. How many are left?*

Multiplication & Division Year 1	
Statutory requirements	**Pupils should be taught to:** ▪ **Solve one step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.** ▪ **Count in multiples of twos, fives and tens** *(Children make connections between arrays, number patterns, and counting in twos, fives and tens).*

Guidance	Through grouping and sharing small quantities, pupils begin to understand: multiplication and division; doubling numbers and quantities; and finding simple fractions of objects, numbers and quantities. They make connections between arrays, number patterns, and counting in twos, fives and tens. Pupils connect halves and quarters to the equal sharing and grouping of sets of objects
Progression	In Year 1, children are introduced to the concepts of multiplication and division, although they will not use the standard signs (× and ÷) until Year 2. In practical activities, using arrays and physical objects such as blocks, children solve multiplication and division problems using small quantities. With support, children investigate the links between arrays, number patterns and their experience of counting in twos, fives and tens.
	Multiplication and Division – Year 1
Representations to support mental and written calculations	Use a range of concrete and pictorial representations, including  4 groups of 3 3 groups of 4  

	 2 + 2 + 2 + 2 + 2 = 10 2 × 5 = 10 2 multiplied by 5 5 pairs 5 hops of 2 	
Mental Strategies	**Regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10.** • Counting in 2s e.g. counting socks, shoes, animal legs... • Counting in 5 s e.g. counting fingers, fingers in gloves, toes … • Counting in 10s e.g. counting fingers,toes... Washing line, and other practical resources for counting. Concrete objects. Numicon; bundles of straws, bead strings. **Memorise and reason with numbers in 2, 5 and 10 times tables**	**Regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10.** Children should be given opportunities to reason about what they notice in number patterns. **Recognise the number of groups counted to support understanding of relationship between multiplication and division.**  2 + 2 + 2 + 2 + 2 = 10 2 × 5 = 10 2 multiplied by 5 5 pairs 5 hops of 2 **Understand division as both sharing and grouping.**

	Represent odd and even numbers. This will help them to understand the pattern in numbers  Understand multiplication as scaling in terms of double and half. (e.g. that tower of cubes is double the height of the other tower) **Understand multiplication is related to doubling and combing groups of the same size (repeated addition**) Recall doubles up to 10. Begin to understand multiplication as scaling in terms of double and half. (e.g. that tower of cubes is double the height of the other tower) **Problem solving with concrete objects (including money and measures)** **Recognise odd and even numbers** Opportunities to reason about what they notice in number patterns.	Sharing – 6 sweets are shared between 2 people. How many do they have each?  Grouping- How many 2's are in 6?  **Use objects to group and share amounts to develop understanding of division in a practical sense.** E.g. using Numicon to find out how many 5's are in 30? How many pairs of gloves if you have 12 gloves? **Explore finding simple fractions of objects, numbers and quantities.** E.g.*16 children went to the park at the weekend. Half that number went swimming. How many children went swimming?* **Vocabulary**

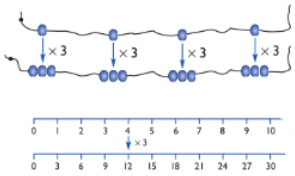
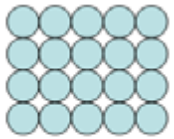
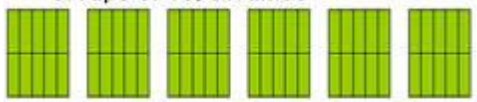
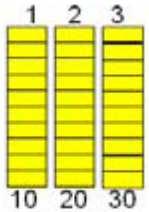
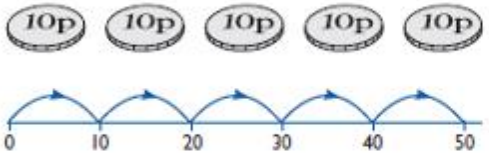
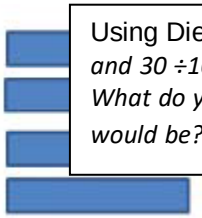
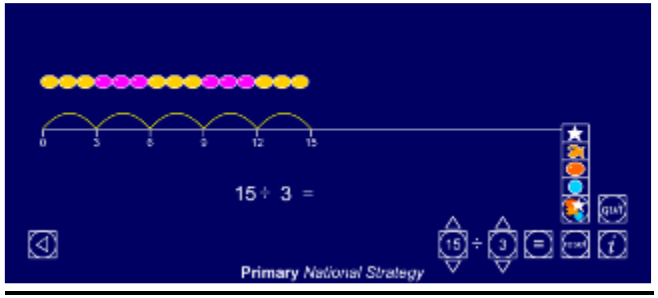
	Write as a number pattern (e.g.5,10,15...;2,4,6...;10,20,30...) Use Cuisenaire and bar method to develop the vocabulary relating to 'times' Pick up five, 4 times **Vocabulary** Ones, groups, lots of, doubling repeated addition groups of, lots of, times, columns, rows longer, bigger, higher etc times as (big, long, wide …etc) **Generalisations** Understand 6 counters can be arranged as 3+3 or 2+2+2 **Some Key Questions** Why is an even number an even number? What do you notice? What's the same? What's different? Can you convince me? How do you know?	• share, share equally, one each, two each…, group, groups of, lots of, array **Generalisations** • True or false? I can only halve even numbers. • Grouping and sharing are different types of problems. Some problems need solving by grouping and some by sharing. Encourage children to practically work out which they are doing. **Some Key Questions** • How many groups of…? • How many in each group? • Share… equally into… • What can do you notice?
Written Calculations	It is important to use a range of models to develop understanding of multiplication, and that children make connections between arrays,	It is important to use a range of models to develop understanding of division and that children make connections between sharing,

	number patterns, and counting in twos, fives and tens. Although there is not statutory requirement for written multiplication in Year 1, we encourage children to begin to write as repeated addition sentences in preparation for Year2 E.g. 2 +2+2+2 =8	grouping, multiplication and division.

Multiplication & Division Year 2	
Statutory requirements	Pupils should be taught to: • count in steps of 2, 3, and 5 from 0, and in tens from any number, forward or backward ▪ recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers

	▪ calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs ▪ show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot ▪ solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.
Guidance	Pupils use a variety of language to describe multiplication and division. Pupils are introduced to the multiplication tables. They practise to become fluent in the 2, 5 and 10 multiplication tables and connect them to each other. They connect the 10 multiplication table to place value, and the 5 multiplication table to the divisions on the clock face. They begin to use other multiplication tables and recall multiplication facts, including using related division facts to perform written and mental calculations. Pupils work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete and continuous quantities, to arrays and to repeated addition. They begin to relate these to fractions and measures (for example, 40 ÷ 2 = 20, 20 is a half of 40). They use commutativity and inverse relations to develop multiplicative reasoning (for example, 4 × 5 = 20 and 20 ÷ 5 = 4).
Progression	In Year 2, children learn the 2, 5 and 10 multiplication tables, and use these facts in calculations. They recognise that multiplication and division have an inverse relationship, and begin to use the × and ÷ symbols. They learn that multiplication is commutative (2 × 10 is the same as 10 × 2) whereas division is not (10 ÷ 2 is not the same as 2 ÷ 10).

	Multiplication	**Division**

Representations to support mental and written calculations	 3 multiplied by 5 ⟶ 3 x 5 3 + 3 + 3 + 3 + 3 =   5 x 4 = 20 Groups of 10, six times  10 x 6 = 60  10 20 30 1 0 x 3 = 30  10p + 10p + 10p + 10p + 10p = 10p × 5 = 50p 5 hops of 10	   Using Dienes: "*If 40÷10=4 and 30 ÷10=3, What do you think 70 ÷ 10 would be? Why?*"  Grouping ITP
Mental Strategies	**Count regularly, on and back, in steps of 2, 3, 5 and 10.**	**Children should count regularly, on and back, in steps of 2, 3, 5 and 10.**

Number lines to be an important image to support thinking.
Practise times table facts
2 x 1 =
2 x 2 =
2 x 3 =
Use a clock face to support understanding of counting in 5s.
Use money to support counting in 2s, 5s, 10s, 20s, 50s

Expressing multiplication as a number sentence using x
Using understanding of the inverse and practical resources to solve missing number problems.

7 x 2 = □ □ = 2 x 7
7 x □ = 14 14 = □ x 7
□ x 2 = 14 14 = 2 x □
□ x = 14 14 = □ x

Understand multiplication using arrays and number lines (see Year 1).
Include multiplications not in the 2, 5 or 10 times tables.
Begin to develop understanding of multiplication as scaling (3 times bigger/taller)
Doubling numbers up to 10 + 10
Link with understanding scaling

Use knowledge to work out other facts such as 2 × 6, 5 × 4, 10 × 9. Show the children how to hold out their fingers and count, touching each finger in turn. So for 2 × 6 (six twos), hold up 6 fingers:

Touching the fingers in turn is a means of keeping track of how far the children have gone in creating a sequence of numbers. The physical action can later be visualised without any actual movement.

This can then be used to support finding out 'How many 3's are in 18?' and children count along fingers in 3's therefore making link between multiplication and division.

Children should continue to develop understanding of division as sharing **and** grouping.

15 pencils shared between 3 pots, how many in each pot?

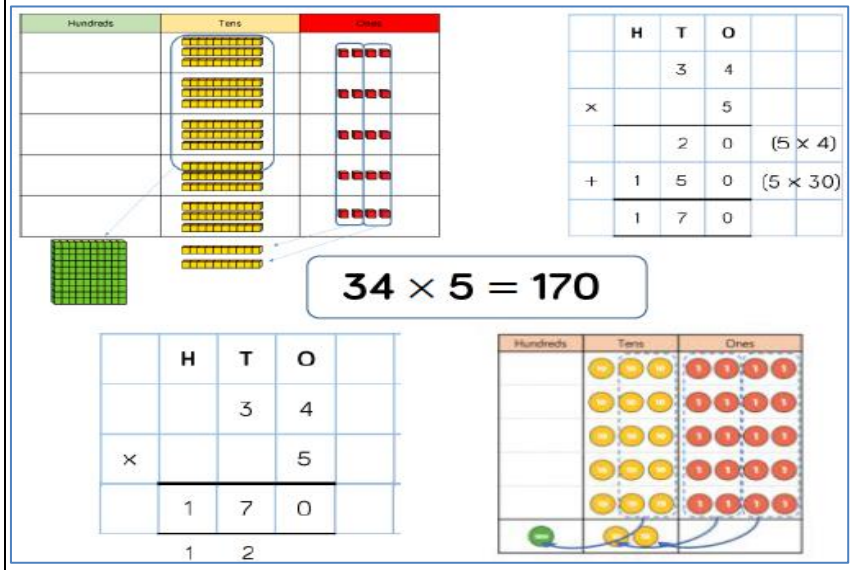
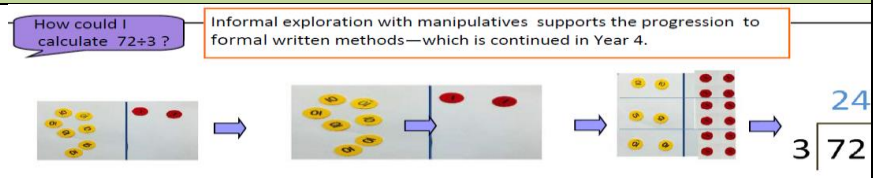
Know and understand sharing and grouping- introducing children to the ÷ sign.
Children should continue to use grouping and sharing for division

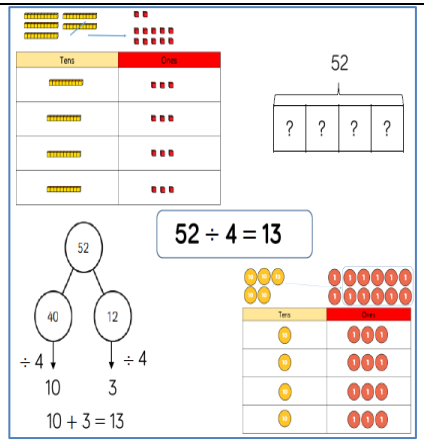
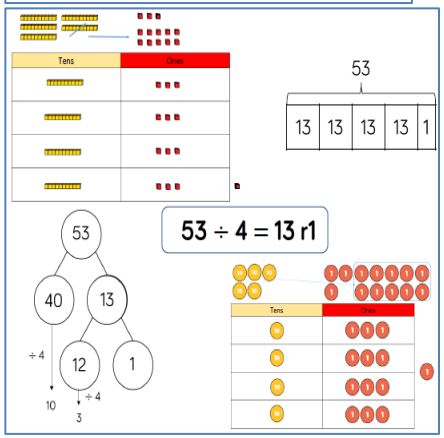
	Instant recall of 2s, 5s, 10s multiplication tables. **Using known doubles to calculate:** (double 15 = double 10 + double 5) **Towards written methods** Use jottings to develop an understanding of doubling two digit numbers. 16 10 6 x2 x2 20 + 12 = **Vocabulary** multiple, multiplication array, multiplication tables / facts groups of, lots of, times, columns, rows **Generalisation** Commutative law shown as an array Repeated addition can be shown mentally on a number line Inverse relationship between multiplication and division. Use an array to explore how numbers can be organised into groups. **Some Key Questions** What do you notice?	using practical apparatus, arrays and pictorial representations. **÷ = signs and missing numbers** 6 ÷ 2 = □ □ = 6 ÷ 2 6 ÷ □ = 3 3 = 6 ÷ □ □ ÷ 2 = 3 3 = □ ÷ 2 □ ÷ ∇ = 3 3 = □ ÷ ∇ **Grouping using a numberline** Group from zero in jumps of the divisor to find our 'how many groups of 3 are there in 15?'. 15 ÷ 3 = 5 Continue work on arrays. Support children to understand how multiplication and division are inverse. Look at an array – what do you see? Children should be given opportunities to find a half, a quarter and a third of shapes, objects, numbers and quantities. Finding a fraction of a number of objects to be related to sharing. They will explore visually and understand how some fractions are equivalent – e.g. two quarters is the same as one half.

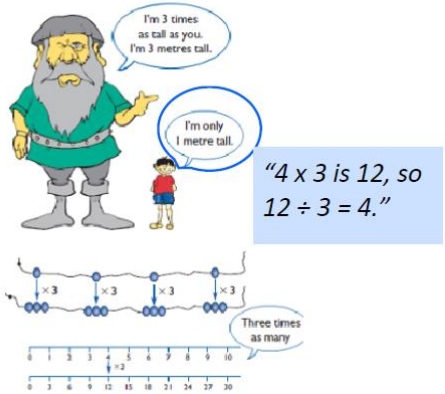
	What's the same? What's different? Can you convince me? How do you know?	Use children's intuition to support understanding of fractions as an answer to a sharing problem. 3 apples shared between 4 people  **Vocabulary** group in pairs, 3s … 10s etc equal groups of divide, ÷, divided by, divided into, remainder **Generalisations** Notice how counting in multiples if 2, 5 and 10 relates to the number of groups you have counted (introducing times tables) An understanding of the more you share between, the less each person will get (e.g. would you prefer to share these grapes between 2 people or 3 people? Why?) Secure understanding of grouping means you count the number of groups you have made. Whereas sharing means you count the number of objects in each group. **Some Key Questions** How many 10s can you subtract from 60?

		I think of a number and double it. My answer is 8. What was my number? If 12 x 2 = 24, what is 24 ÷ 2? Questions in the context of money and measures (e.g. how many 10p coins do I need to have 60p? How many 100ml cups will I need to reach 600ml?)
Written	**Expressing multiplication as a number sentence using x** **Using understanding of the inverse and practical resources to solve missing number problems.**	**Expressing division as a number sentence using ÷ and = signs solving problems with missing numbers.**

Multiplication & Division Year 3	
Statutory requirements	Pupils should be taught to: ▪ recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables ▪ write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods ▪ solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.
Guidance	Pupils continue to practise their mental recall of multiplication tables when they are calculating mathematical statements in order to improve fluency. Through doubling, they connect the 2, 4 and 8 multiplication tables. Pupils develop efficient mental methods, for example, using commutativity and associativity (for example, 4 × 12 × 5 = 4 × 5 × 12 = 20 × 12 = 240) and multiplication and division facts (for example, using 3 × 2 = 6, 6 ÷ 3 = 2 and 2 = 6 ÷ 3) to derive related facts (for example, 30 × 2 = 60, 60 ÷ 3 = 20 and 20 = 60 ÷ 3). Pupils develop reliable written methods for multiplication and division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written methods of short multiplication and division. Pupils solve simple problems in contexts, deciding which of the four operations to use and why. These include measuring and scaling contexts, (for example, four times as high, eight times as long etc.) and correspondence problems in which m objects are connected

	to n objects (for example, 3 hats and 4 coats, how many different outfits?; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children).	
Progression	In Year 3, children learn the 3, 4 and 8 multiplication tables, and use their knowledge of doubling to explore links between the 2, 4 and 8 multiplication tables. They use facts from these new multiplication tables to solve multiplication and division problems. Building on their work with written mathematical statements in Year 2, they begin to develop more formal written methods of multiplication and division. They will extend this in Year 4 when they work with more complex multiplication and division problems.	
	Multiplication	**Division**
Representations to support mental and written calculations	Use a range of concrete and  pictorial representations, including: • Teachers may decide to look at the expanded column method before moving on to the short multiplication	

	method, if they feel children need it. • The place value counters should support the understanding of the method rather than the multiplication, as children should use times table knowledge.	  • When dividing numbers involving an exchange, children can use base 10 and place value counters to exchange 1 ten for 10 ones. Children should start with the equipment outside the place value grid, before sharing the tens and ones equally between the rows.

	Multiplication	Division
Mental Calculations	• recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables (and 2, 5 and 10 multiplication tables from Y2) • Use doubling to connect 2, 4 and 8 multiplication tables • Develop efficient mental methods using commutativity and associativity • Derive related multiplication and division facts • calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental methods • Partitioning: multiply the tens first and then multiply the units, e.g. 57 x 6 = (50 x 6) + (7 x 6) = 300 + 42 = 342 Ensure opportunities to learn multiplication tables through use of visual models, images and also rote learning. **The associative law:** 4 x 12 x 5 = 4 x 5 12 = 20 x 12 = 240 **The commutative law:** 4 x 12 = 12 x 4 **Deriving related facts:** 3 x 2 = 60, 6 ÷ 3 = 2, 6 ÷ 2 = 3 ➡ 30 x 2 = 60, 60 ÷ 3 = 20, 20 = 60 ÷ 3	Pupils should be taught to recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables. Pupils continue to practise their mental recall of multiplication tables in order to improve fluency. Pupils derive division and multiplication facts and gain an understanding of the relationship between the two. 36 ÷3 = 12 30 6 30 ÷3=10 6 ÷3=2 +  *"4 x 3 is 12, so 12 ÷ 3 = 4."*

<table>
<tr>
<td>Written Calculations</td>
<td>Teachers may decide to look at the expanded column method before moving on to the short multiplication method, if they feel children need it.
The place value counters should support the understanding of the method rather than the multiplication, as children should use times table knowledge.

<table>
<tr><td></td><td>H</td><td>T</td><td>O</td><td></td><td></td></tr>
<tr><td></td><td></td><td>3</td><td>4</td><td></td><td></td></tr>
<tr><td>×</td><td></td><td></td><td>5</td><td></td><td></td></tr>
<tr><td></td><td></td><td>2</td><td>0</td><td>(5 × 4)</td><td></td></tr>
<tr><td>+</td><td>1</td><td>5</td><td>0</td><td>(5 × 30)</td><td></td></tr>
<tr><td></td><td>1</td><td>7</td><td>0</td><td></td><td></td></tr>
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<table>
<tr><td></td><td>H</td><td>T</td><td>O</td><td></td></tr>
<tr><td></td><td></td><td>3</td><td>4</td><td></td></tr>
<tr><td>×</td><td></td><td></td><td>5</td><td></td></tr>
<tr><td></td><td>1</td><td>7</td><td>0</td><td></td></tr>
<tr><td></td><td>1</td><td>2</td><td></td><td></td></tr>
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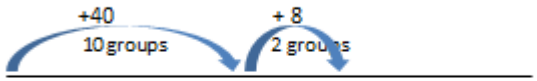
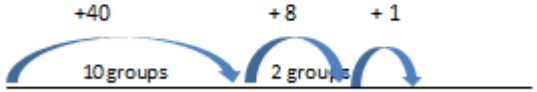
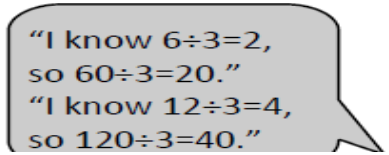
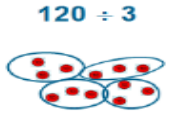
</td>
<td>Becoming more efficient using a numberline

Children need to be able to partition the dividend in different ways.
48 ÷ 4 = 12
 

Remainders
49 ÷ 4 = 12 r1
 

Sharing – 49 shared between 4. How many left over?
Grouping – How many 4s make 49. How many are left over?



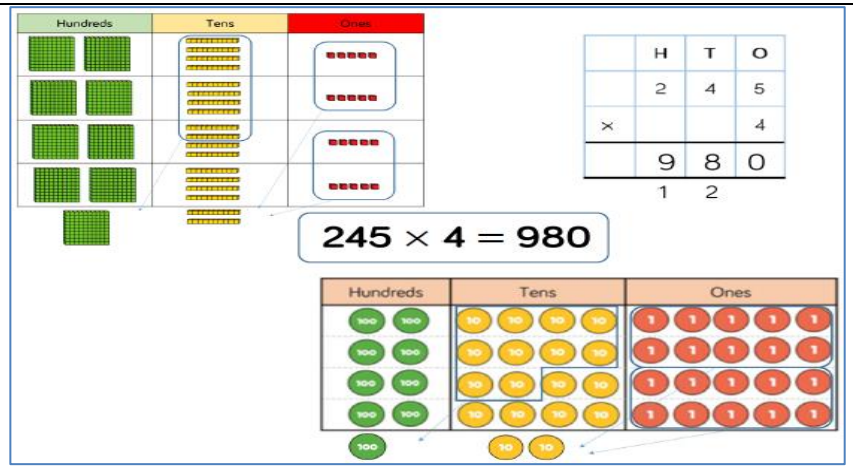
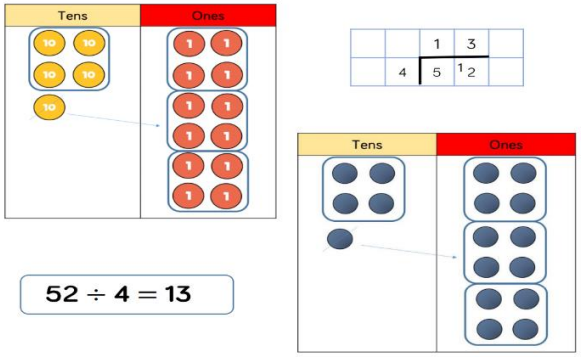
New written methods can be modelled alongside concrete or pictorial representations to ensure understanding.</td>
</tr>
<tr>
<td>Vocabulary</td>
<td>multiple, multiplication array, multiplication tables / facts groups of, lots of, times, columns, rows, partition, product, multiples of 4, 8, 50 and one hundred, scale up, commutative</td>
<td>group in pairs, 3s … 10s etc, equal groups of, divide, ÷, divided by, divided into, remainder, inverse, non-commutative</td>
</tr>
</table>

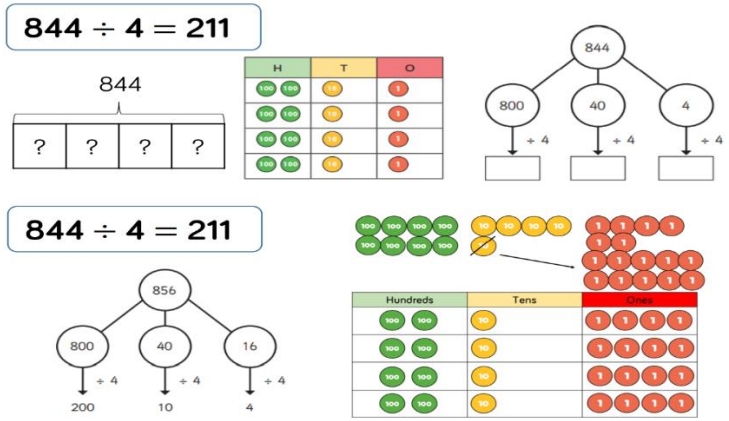
Generalisations	Connecting x2, x4 and x8 through multiplication facts Comparing times tables with the same times tables which is ten times bigger. If 4 x 3 = 12, then we know 4 x 30 = 120. Use place value counters to demonstrate this. When they know multiplication facts up to x12, do they know what x13 is? (i.e. can they use 4x12 to work out 4x13 and 4x14 and beyond?) **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?	Inverses and related facts – develop fluency in finding related multiplication and division facts. Develop the knowledge that the inverse relationship can be used as a checking method. **Some Key Questions** Questions in the context of money and measures that involve remainders (e.g. How many lengths of 10cm can I cut from 81cm of string? You have £54. How many £10 teddies can you buy?) What is the missing number? 17 = 5 x 3 + __ __ = 2 x 8 + 1

Multiplication & Division Year 4

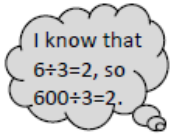
Statutory requirements	Pupils should be taught to: ▪ recall multiplication and division facts for multiplication tables up to 12 × 12 ▪ use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers ▪ recognise and use factor pairs and commutativity in mental calculations ▪ multiply two-digit and three-digit numbers by a one-digit number using formal written layout ▪ solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.
Guidance	Pupils continue to practise recalling and using multiplication tables and related division facts to aid fluency. Pupils practise mental methods and extend this to three-digit numbers to derive facts, (for example 600 ÷ 3 = 200 can be derived from 2 x 3 = 6).
Progression	● Children should continue to practise recalling and using multiplication tables and related division facts to aid fluency. ● Children should practise mental methods and extend this to three-digit numbers to derive facts, for example 200 × 3 = 600 into 600 ÷ 3 = 200. ● Children should practise to become fluent in the formal written method of short multiplication for multiplying using multi-digit numbers, and short division with exact answers when dividing by a one- digit number (see Appendix 1). ● Children should write statements about the equality of expressions (e.g. use the distributive law 39 × 7 = 30 × 7 + 9 × 7 and associative law (2 × 3) × 4 = 2 × (3 × 4)). They combine their knowledge of number facts and rules of arithmetic to solve mental and written calculations, e.g. 2 × 6 × 5 = 10 × 6. ● Children should solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as the number of choices of a meal on a menu, or three cakes shared equally between 10 children.

	Multiplication	**Division**

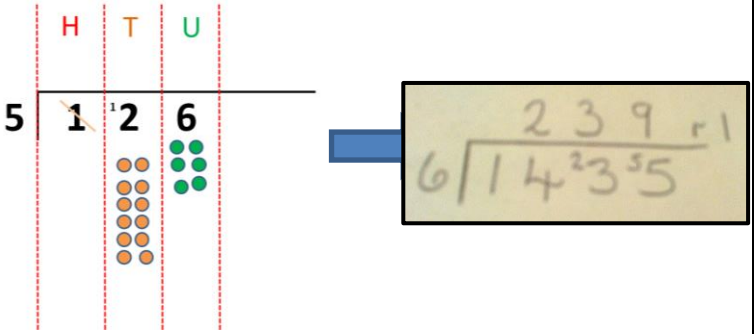
Representations to support mental and written calculations	 • When moving to a 3 digit by a 1 digit multiplication, encourage children to use the short formal written method. • Base 10 and place value counters continue to support the understanding of the written method. Limit the number of exchanges needed in the questions and move children away from resources when multiplying larger numbers.	**Division by grouping**  **Division by grouping** – When using the short division method, children use grouping. Starting with the largest place value, they group by the divisor. Language is important here. Children should consider "How many groups of 4 tens can we make?" and "how many groups of 4 ones can we make?" Remainders can also be seen as they are left ungrouped. **Division by sharing** – Children can continue to use place value counters to share 3 digit numbers into equal groups.

		Children should start with the equipment outside the place value grid before sharing the hundreds, tens and ones equally between the rows. Flexible partitioning in a whole part model supports this method. **Division by sharing** 

	Multiplication	Division

Mental Calculations	• Recall multiplication and division facts for tables up to 12 x 12
	• Use place value, known and derived facts to multiply and divide mentally, including:
	  • Multiplying by 0 and 1
	  • Dividing by 1
	  • Multiplying together 3 numbers, eg 2 x 6 x 5 = 10 x 6 = 60
	• Recognise and use factor pairs and commutativity in mental calculations
	• Practise mental methods and extend this to three-digit numbers to derive facts, eg 600 ÷ 3 = 200 can be derived from 2 x 3 = 6
	
	Using the **distributive** law:
	39 x 7 = 30 x 7 + 9 x 7
	Using the **associative** law:
	(2 x 3) x 4 = 2 x (3 x 4)
	
	Using facts and rules:
	2 x 6 x 5 = 10 x 6 = 60
	
	Counting in multiples of 6, 7, 9, 25 and 1000, and steps of 1/100. Solving practical problems where children need to scale up. Relate to known number facts. (e.g. how tall would a 25cm sunflower be if it grew 6 times taller?)
		Pupils should be taught to:
		• recall multiplication and division facts for multiplication tables up to 12 × 12
		• use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
		• recognise and use factor pairs and commutativity in mental calculations
		
		 
		
		Pupils practise mental methods and extend this to three-digit numbers to derive facts.
		Children should experience regular counting on and back from different numbers in multiples of 6, 7, 9, 25 and 1000.
		Children should learn the multiplication facts to 12 x 12.

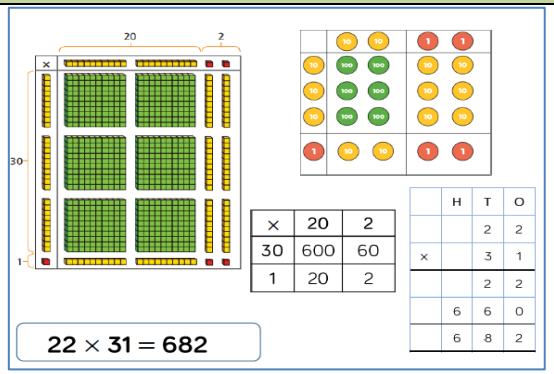
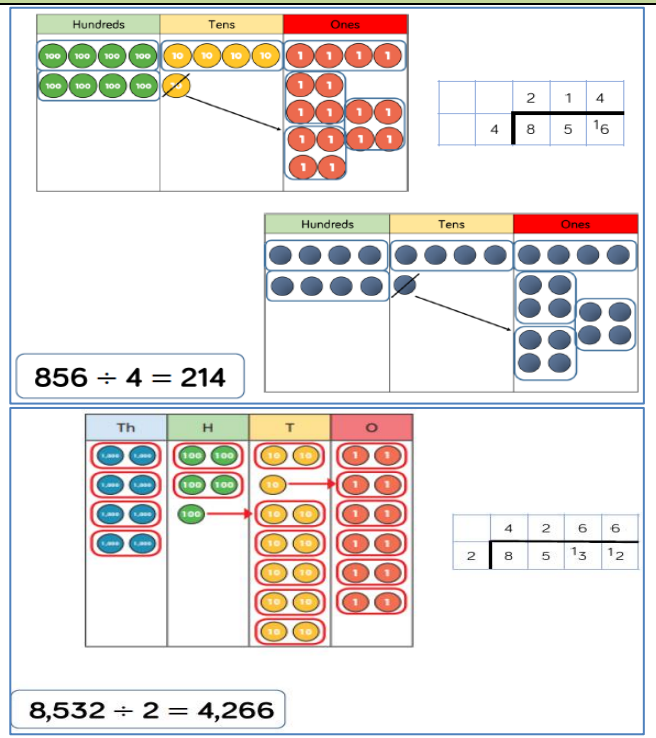
Written Calculations	Children to embed and deepen their understanding of the grid method to multiply up 2d x 2d. Ensure this is still linked back to their understanding of arrays and place value counters. See the visual representations above.	Formal short division should only be introduced once children have a good understanding of division, its links with multiplication and the idea of 'chunking up' to find a target number (see use of number lines above)

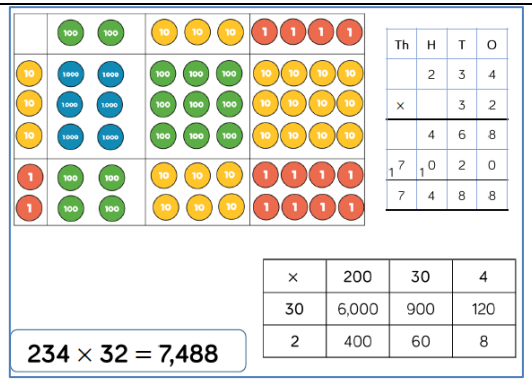
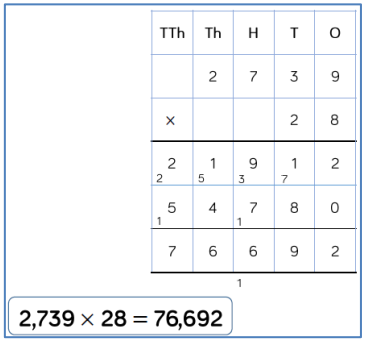
	• Multiply two-digit and three-digit numbers by a one digit number using formal written layout • Estimate before calculating • Ensure written methods build on / relate to mental methods Key skills to support: • Know or quickly recall multiplication facts up to 12 x 12 • Understand the effect of multiplying by 10, 100 or 1000 • Multiply multiples of 10, eg 20 x 40 • Approximate, eg recognise that 72 x 38 is approximately equal 70 x 40 = 2800 and use this information to check whether answers are sensible.	Short division to be modelled for understanding using place value counters as shown below. Calculations with 2 and 3-digit dividends. E.g. fig 1 
Vocabulary	multiple, multiplication array, multiplication tables / facts groups of, lots of, times, columns, rows, partition, product, multiples of 4, 8, 50 and one hundred, scale up, commutative, multiplication facts up to 12 x 12, derive.	divide, divided by, divisible by, divided into share between, groups of factor, factor pair, multiple times as (big, long, wide …etc) equals, remainder, quotient, divisor inverse
Generalisations	Children given the opportunity to investigate numbers multiplied by 1 and 0. When they know multiplication facts up to x12, do they know what x13	**Generalisations** True or false? Dividing by 10 is the same as dividing by 2 and then dividing by 5. Can you find any more rules like this?

	is? (i.e. can they use 4x12 to work out 4x13 and 4x14 and beyond?) **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?	Is it sometimes, always or never true that □ ÷ Δ = Δ ÷ □? Inverses and deriving facts. 'Know one, get lots free!' e.g.: 2 x 3 = 6, so 3 x 2 = 6, 6 ÷ 2 = 3, 60 ÷ 20 = 3, 600 ÷ 3 = 200 etc. Sometimes, always, never true questions about multiples and divisibility. (When looking at the examples on this page, remember that they **may not** be 'always true'!) E.g.: • Multiples of 5 end in 0 or 5. • The digital root of a multiple of 3 will be 3, 6 or 9. • The sum of 4 even numbers is divisible by 4.

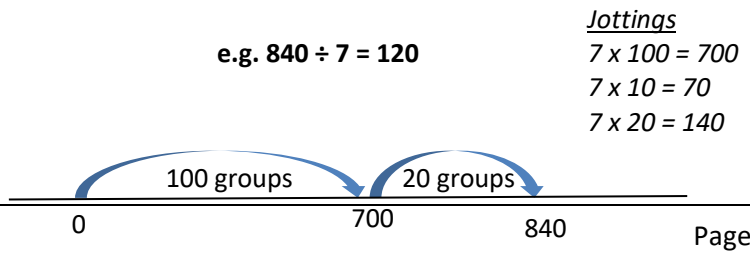
Multiplication & Division Year 5	
Statutory requirements	Pupils should be taught to: ▪ identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers ▪ know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers ▪ establish whether a number up to 100 is prime and recall prime numbers up to 19 ▪ multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

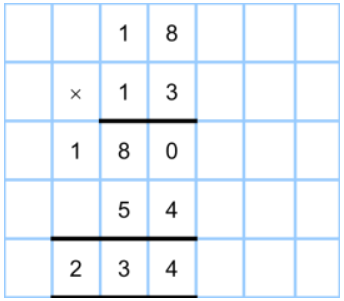
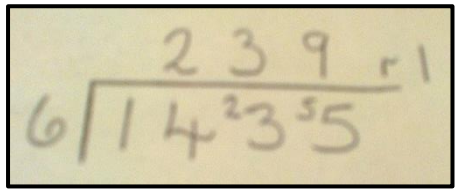
	▪ multiply and divide numbers mentally drawing upon known facts ▪ divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context ▪ multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 ▪ recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3}) ▪ solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes ▪ solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign ▪ solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.
Guidance	Pupils practise and extend their use of the formal written methods of short multiplication and short division (see Mathematics Appendix 1). They apply all the multiplication tables and related division facts frequently, commit them to memory and use them confidently to make larger calculations. They use and understand the terms factor, multiple and prime, square and cube numbers. Pupils interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (for example, 98 ÷ 4 = = 24 r 2 = 24 = 24.5 ≈ 25). 4 98 2 1 Pupils use multiplication and division as inverses to support the introduction of ratio in year 6, for example, by multiplying and dividing by powers of 10 in scale drawings or by multiplying and dividing by powers of a 1000 in converting between units such as kilometres and metres. Distributivity can be expressed as $a(b + c) = ab + ac$. They understand the terms factor, multiple and prime, square and cube numbers and use them to construct equivalence statements (for example, 4 x 35 = 2 x 2 x 35; 3 x 270 = 3 x 3 x 9 x 10 = 9^2 x 10). Pupils use and explain the equals sign to indicate equivalence, including in missing number problems (for example, 13 + 24 = 12 + 25; 33 = 5 x□).
Progression	• Children should practise and extend their use of the formal written methods of short multiplication and division (see National Curriculum Appendix 1). They apply all the multiplication tables and related division facts, commit them to memory and use them confidently to make larger calculations.

	• They should use and understand the terms factor, multiple and prime, square and cube numbers. • Children should interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding • Children use multiplication and division as inverses to support the introduction of ratio in Year 6, by multiplying and dividing by powers of 10 in scale drawings or by multiplying and dividing by powers of a 1000 in converting between units such as kilometres and metres. Distributivity can be expressed as a(b +c) = ab + ac in preparation for using algebra.	
	Multiplication	**Division**
Representations to support mental and written calculations		 • Place value counters or plain counters can be placed on a place value grid to support children to divide 4 digits

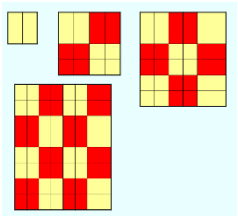
	  • When multiplying a multi digit number by 2 digits, use the area model to help children understand the size of the numbers they're using. • The grid method matches the area model as an initial written method before moving on to the formal written method.	by 1 digit. Children can also draw their own counters and group them through a more pictorial method. • Children should be encouraged to move away from the concrete and pictorial when dividing numbers with multiple exchanges.

	• When children are multiplying 4 digits by 2 digits, children should be confident in the formal written method.	

	Multiplication	Division
Mental Calculations	Children should continue to count regularly, on and back, now including steps of powers of 10. The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate. Children should continue to partition numbers in different ways.	Children should count regularly using a range of multiples, and powers of 10, 100 and 1000, building fluency. Children should practice and apply the multiplication facts to 12 x 12. 

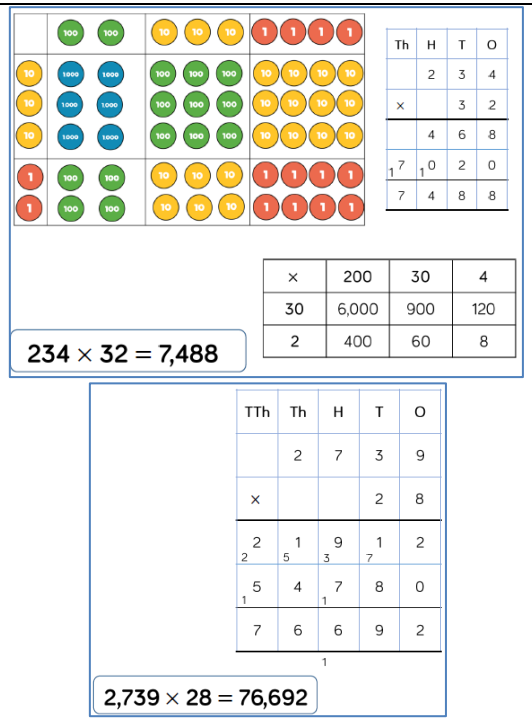
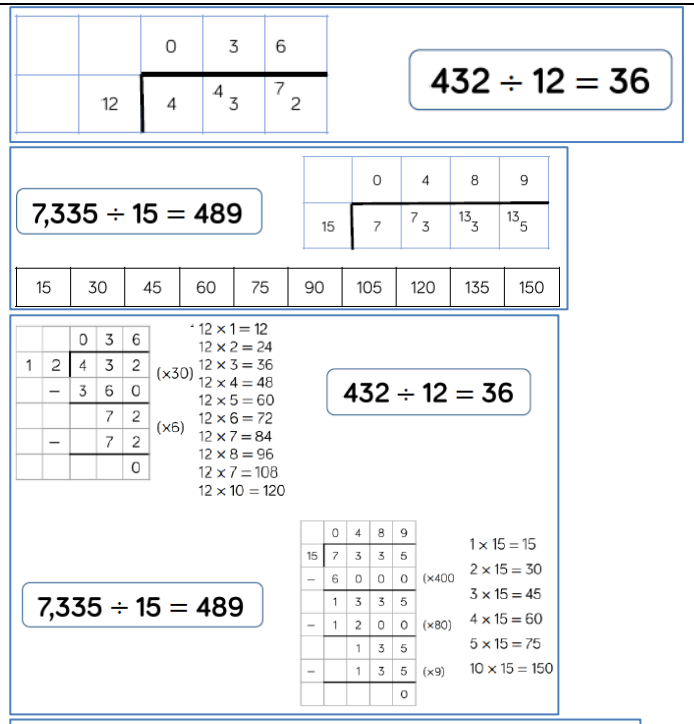
	X by 10, 100, 1000 using moving digits ITP Use practical resources and jottings to explore equivalent statements (e.g. 4 x 35 = 2 x 2 x 35) Recall of prime numbers up 19 and identify prime numbers up to 100 (with reasoning) Solving practical problems where children need to scale up. Relate to known number facts. Identify factor pairs for numbers	
Written Calculations	Children continue to practise using an efficient formal method of multiplication:  	<u>Formal Written Methods</u> Continued as shown in Year 4, leading to the efficient use of a formal method. The language of grouping to be used (see link from fig. 1 in Year 4) E.g. 1435 ÷ 6  Children begin to practically develop their understanding of how express the remainder as a decimal or a fraction. Ensure practical understanding allows children to work through this

		(e.g. what could I do with this remaining 1? How could I share this between 6 as well?)
Vocabulary	multiple, multiplication array, multiplication tables / facts groups of, lots of, times, columns, rows, partition, product, multiples of 4, 8, 50 and one hundred, scale up, commutative, multiplication facts up to 12 x 12, derive, cube numbers, prime numbers, square numbers, common factors, prime factors, composite numbers, formal written method	divide, divided by, divisible by, divided into, share between, groups of factor, factor pair, multiple, times as (big, long, wide …etc) equals, remainder, quotient, divisor, inverse, common factors, prime number, prime factors, composite numbers, short division, square number, cube number inverse, power of
Generalisations	Relating arrays to an understanding of square numbers and making cubes to show cube numbers. Understanding that the use of scaling by multiples of 10 can be used to convert between units of measure (e.g. metres to kilometres means to times by 1000) **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?	The = sign means equality. Take it in turn to change one side of this equation, using multiplication and division, e.g. Start: **24 = 24** Player 1: **4 x 6 = 24** Player 2: **4 x 6 = 12 x 2** Player 1: **48 ÷ 2 = 12 x 2** Sometimes, always, never true questions about multiples and divisibility. E.g.:

	How do you know this is a prime number?	• If the last two digits of a number are divisible by 4, the number will be divisible by 4.  • If the digital root of a number is 9, the number will be divisible by 9. • When you square an even number the result will be divisible by 4 (one example of 'proof' shown left)

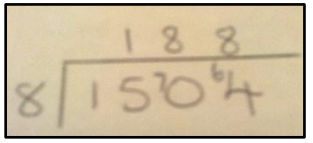
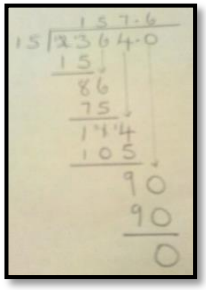
Multiplication & Division Year 6	
Statutory requirements	Pupils should be taught to: ▪ multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication ▪ divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context ▪ divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context

	▪ perform mental calculations, including with mixed operations and large numbers ▪ identify common factors, common multiples and prime numbers ▪ use their knowledge of the order of operations to carry out calculations involving the four operations
Guidance	Pupils practise addition, subtraction, multiplication and division for larger numbers, using the formal written methods of columnar addition and subtraction, short and long multiplication, and short and long division (see Mathematics Appendix 1). They undertake mental calculations with increasingly large numbers and more complex calculations. Pupils continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency. Pupils round answers to a specified degree of accuracy, for example, to the nearest 10, 20, 50 etc., but not to a specified number of significant figures. Pupils explore the order of operations using brackets; for example, 2 + 1 x 3 = 5 and (2 + 1) x 3 = 9. Common factors can be related to finding equivalent fractions.
Progression	● Children should practise addition, subtraction, multiplication and division for larger numbers, using the efficient written methods of columnar addition and subtraction, short and long multiplication, and short and long division (see Appendix 1). ● They should undertake mental calculations with increasingly large numbers and more complex calculations. ● Children should continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency. ● Children should round answers to a specified degree of accuracy. ● Children explore the order of operations using brackets. ● Common factors can be related to finding equivalent fractions.
	Multiplication | **Division**

Representations to support mental and written calculations	 • When multiplying a multi digit number by 2 digits, use the area model to help children understand the size of the numbers they're using. • The grid method matches the area model as an initial written method before moving on to the formal written method.	

	When children are multiplying 4 digits by 2 digits, children should be confident in the formal written method.	

	Multiplication	Division
Mental Calculations	Children should experiment with order of operations, investigating the effect of positioning the brackets in different places, e.g. 20 – 5 x 3 = 5; (20 – 5) x 3 = 45 They should be encouraged to choose from a range of strategies to solve problems mentally: - Partitioning using x10, x20 etc - Doubling to solve x2, x4, x8 - Recall of times tables - Use of commutativity of multiplication If children know the times table facts to 12 x 12. Can they use this to recite other times tables (e.g. the 13 times tables or the 24 times table)	**Mental Strategies** Consolidate previous years. Children should experiment with order of operations, investigating the effect of positioning the brackets in different places, e.g. 20 – 5 x 3 = 5; (20 – 5) x 3 = 45

Written Calculations	Continue to refine and deepen understanding of written methods including fluency for using long multiplication 2 3 1 1342 x 18 13420 10736 24156 1	**÷ = signs and missing numbers** Continue using a range of equations but with appropriate numbers **Sharing and Grouping and using a number line** Children will continue to explore division as sharing and grouping, and to represent calculations on a number line as appropriate.   Quotients should be expressed as decimals and fractions **Formal Written Methods – long and short division** E.g. 1504 ÷ 8 E.g. 2364 ÷ 15
Vocabulary	multiple, multiplication array, multiplication tables / facts groups of, lots of, times, columns, rows, partition, product, multiples of 4, 8, 50 and one hundred, scale up, commutative, multiplication facts up to 12 x 12, derive, cube numbers, prime numbers, square numbers, common factors, prime factors, composite numbers, formal	divide, divided by, divisible by, divided into, share between, groups of factor, factor pair, multiple, times as (big, long, wide …etc) equals, remainder, quotient, divisor, inverse, common factors, prime number, prime factors, composite numbers, short division, square number, cube number inverse, power of

	written method, common multiples, order of operations	
Generalisations	Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as BODMAS, or could be encouraged to design their own ways of remembering. Understanding the use of multiplication to support conversions between units of measurement. **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?	Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as BODMAS, or could be encouraged to design their own ways of remembering. Sometimes, always, never true questions about multiples and divisibility. E.g.: If a number is divisible by 3 and 4, it will also be divisible by 12. (also see year 4 and 5, and the hyperlink from the Y5 column) Using what you know about rules of divisibility, do you think 7919 is a prime number? Explain your answer.

		<u>Some Key Questions for Year 4 to 6</u> **What do you notice?** **What's the same? What's different?** **Can you convince me?** **How do you know?**